Overshooting and the Exchange Rate Disconnect Puzzle : A Reappraisal

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Le sur-ajustement du taux de change et l'énigme de la déconnexion du taux de change : une ré-évaluation

Résumé : Depuis le début des années 70, la période des changes flexibles a été associée à une forte volatilité des taux de change nominaux et réels, sans modification significative de la distribution des variables macroéconomiques fondamentales. Selon Dornbusch [1976], cette déconnexion entre la volatilité du taux de change nominal et celle de ses fondamentaux serait liée au sur-ajustement du taux de change. Lorsque la parité des taux d'intérêt non couverte est vérifiée, le sur-ajustement du taux de change provient d'un différentiel négatif et persistant entre les taux d'intérêt national et étranger. Nous évaluons la pertinence empirique de ce mécanisme dans le cadre d'un modèle de participation limitée étendu à un cadre international. L'introduction de coûts d'ajustement sur la modification des encaisses monétaires permet d'accroître l'ampleur du sur-ajustement du taux de change. Ce dernier apparaît comme un phénomène important dans l'explication de la forte volatilité du taux de change nominal.

Mots clés : Déconnexion du taux de change, sur-ajustement, effet liquidité, parité des taux d'intérêt non couverte

Overshooting and the Exchange Rate Disconnect Puzzle : A Reappraisal

Abstract : Transition to floating exchange rate regimes has led to sharp increases in nominal and real exchange rate volatilities with no corresponding changes in the distribution of fundamental macroeconomic variables. In the spirit of Dornbusch [1976], we assess whether nominal exchange rate overshooting is responsible for this phenomenon. As long as uncovered interest rate parity holds, nominal exchange rate overshooting is linked to a persistent fall in the spread between domestic and foreign nominal interest rates. We thus develop a limited participation model in an international setting. Introducing adjustment costs on money holdings in the limited participation framework substantially raises the magnitude of the overshooting dynamics. Overshooting indeed plays a key role in understanding the extreme nominal exchange rate volatility.

Keywords : Exchange rate disconnect puzzle, overshooting, liquidity effect, uncovered interest rate parity

Codes JEL: F31, F41

Contents

1 Introduction

Why are the exchange rates so volatile and so apparently disconnected from fundamentals ? Obstfeld and Rogoff [2000] have recently underlined the difficulty of convincingly answering to the so-called exchange rate disconnect puzzle.

> *"Exchange rates are remarkably volatile relative to any model we have of underlying fundamentals such as interest rates, outputs and money supplies and no model seems to be very good at explaining exchange rates even ex-post."* Obstfeld and Rogoff [2000].

Much recent work in the international business cycle literature has been trying to explain this disconnect puzzle. Although an obvious starting point for solving this puzzle could have been the popular Dornbusch overshooting story, it is not at the heart of the Chari, Kehoe and McGrattan [2001]'s model which is the most advanced work in this literature. They develop a complex two-country model allowing for price discrimination and staggered price-setting. Given the price stickiness, real and nominal exchange rates are highly correlated and the mechanisms of their model imply that real exchange rate volatility is linked to the risk aversion parameter and the volatility of consumption. They obtain theoretical nominal and exchange rates volatilities that fit the data quite well. However, in order to obtain such results, they impose specific assumptions namely separable preferences, high risk aversion and price stickiness of at least one year, assumptions which appear somewhat unappealing.

An alternative way of accounting for the disconnect puzzle could indeed focus on Dornbusch [1976]'s overshooting story. An expansionary monetary shock would generate a persistent fall in the nominal interest rate. As long as uncovered interest rate parity holds, this persistent decline in the spread between domestic and foreign interest rates results in a nominal exchange rate overshooting.

The ongoing research on the VAR methodology has offered numerous works on the empirical relevancy of that point, even if at first the results seem at odds with the overshooting hypothesis. In a seminal paper, Eichenbaum and Evans [1995] have shown that, in response to a tighter US monetary policy, the US dollar exhibits a delayed overshooting pattern of 2 to 3 years *vis-à-vis* the major currencies. Positive interest differentials are associated with persistent appreciation of the US dollar. This delayed overshooting is not consistent with the Dornbusch story and leads to the so-called *forward discount bias puzzle*: the forward exchange rate is a biased predictor of the future spot rate.

Lack of accuracy in the measurement of monetary policy shocks may help explain why exchange rates do not exhibit any overshooting path. For instance, Bonser-Neal, Vance Roley and Sellon [1998] use the federal funds rate target, rather than the actual rate, as a proxy for monetary policy changes. Using an event-study methodology instead of the typical VAR approach, they show that the overshooting hypothesis appears to be consistent with the data for all major currencies except the Yen / Dollar exchange rate. Similarly, Kalyvitis and Michaelides [2001] re-examine the impact of US monetary policy shocks on exchange rate using the monetary policy indicator proposed by Bernanke and Mihov [1998]. They find evidence for instantaneous, rather than delayed, overshooting after a monetary shock when relative output and relative prices are included in a VAR specification.

In addition, some recent developments of the VAR methodology have led to reconcile the facts with a traditional overshooting story. First, Faust and Rogers [2000] apply Faust [1998]'s identification techniques to the US/UK and US/DM exchange rate dynamics. They find that the delayed overshooting result is quite sensitive to dubious identifying assumptions[1]. Kim and Roubini [2000]'s results go one-step further in favor of the overshooting story. Unlike Eichenbaum and Evans [1995], they use a structural VAR approach with non-recursive contemporaneous restrictions, in the lines of Sims and Zha [1998]. They aim at a more careful identification of the true innovation in the monetary supply process. They get that, initially the nominal exchange rate appreciates in response to a monetary contraction; after a few months, instead of the long and persistent appreciation found in Eichenbaum and Evans [1995], it depreciates over time in accordance with the uncovered interest rate parity condition. Kim and Roubini [2000] particularly investigate this last point. According to the uncovered parity implication, the conditional expectation of the excess return must be equal to zero. On the basis of their estimated VAR model, they compute the impulse response function of this expectation to a monetary policy shock. They do not find systematic evidence of significant excess returns. Thus, the uncovered interest rate parity holds in the short run. Moreover, they find that monetary policy shocks explain a very large share of the nominal exchange rate short-run variations.

All these results lead us to come back to the traditional overshooting explanation given by Dornbusch [1976] in order to quantitatively verify in a general equilibrium setting that this phenomenon is an essential part of the high observed volatility of nominal exchange rate. Our objective is thus twofold. From a qualitative point of view, we aim at understanding how monetary policy can lead to an overshooting of nominal exchange rate in a business cycle style model. Secondly, our goal is to check whether this transmission channel has quantitative implications in generating some disconnect between the nominal exchange rate and its macroeconomic fundamentals.

A limited participation international business cycle model is then developed. Indeed, the limited participation assumption pioneered by Christiano [1991] and Fuerst [1992] aims at reproducing the persistent fall in the nominal interest rate following a monetary expansion in a closed economy setting: this assumption states that the household decides the amount of money she wants to put into the banks before the occurrence of the monetary injection. Following Dow [1995], we assume that firms have to borrow in order to invest in physical capital. Because of the specific path of investment induced by a positive monetary shock, this assumption allows the limited participation model to generate a liquidity effect. Furthermore,

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[1] However, large deviations from the uncovered interest parity remain unexplained.

it is well-known that a convenient way to generate a large and persistent liquidity effect consists in assuming adjustment costs on money holdings (Christiano and Eichenbaum [1992], King and Watson [1996]). The liquidity effect has been already analyzed in an open economy setting by Schlagenhauf and Wrase [1995]. Yet the focus of their paper is quite different from ours. The authors aim at reproducing the dynamic responses of the interest rate, the nominal exchange rate and the output given by a structural VAR model, following a monetary expansion. By using a two-country framework, Schlagenhauf and Wrase [1995] also measure the role played by the liquidity effect in the international transmission of economic fluctuations. In contrast, our paper highlights the crucial role played by the overshooting phenomenon in explaining the nominal exchange rate volatility.

The streamlined framework we propose highlights the specific channels through which monetary policy affects the interest rate and the nominal exchange rate. We adopt the small open economy assumption: since it abstracts from movements in foreign interest rate or inflation, it allows us to identify the domestic propagation mechanisms in a very transparent way. As in Dornbusch [1976], the nominal exchange rate overshooting results from rigidities on the market where the interest rate is determined. In Dornbusch [1976]'s framework, the nominal interest rate is determined by a LM equation. Price stickiness thus magnifies the interest rate response to monetary shocks. In this paper, predetermined bank deposits combined with adjustment costs on money holdings constitute the sources of rigidities that generate a persistent fall in interest rate following nominal impulses.

In order to assess the empirical relevance of the overshooting phenomenon, the following strategy is adopted: for a given volatility of the money supply, we compare the nominal exchange rate volatility generated by two models that differ from the exchange rate dynamics.

Firstly, a cash-in-advance economy is considered as a benchmark model to evaluate the quantitative implications of overshooting dynamics relative to a standard monotonic adjustment path model. Indeed, a generic implication of the cash-in-advance model is to provoke an increase in the spread between domestic and foreign interest rates following a positive monetary shock due to a Fisher effect. Given uncovered interest rate parity, the nominal exchange rate then displays a monotonic depreciation along the adjustment path to its new steady state value. As a result, the model lacks amplification mechanisms : the volatility of the nominal exchange rate is close to that of its monetary fundamental.

Secondly, we evaluate whether the overshooting phenomenon is able to amplify the exogenous money supply shocks. The limited participation model with adjustment costs on money holdings thus accounts for a persistent liquidity effect after a positive monetary shock and a nominal exchange rate overshooting. We then show that the overshooting intensity determines the ability of the model to replicate the observed volatility of the nominal exchange rate.

The paper is organized as follows. Section 2 presents the building blocks of the basic cash-in-advance model. The intrinsic failure of this kind of model (whatever the source of borrowing, labor or investment) to reproduce the implications of a monetary shock drives us to develop a limited participation model with adjustment costs on money holdings in section 3. Section 4 concludes.

2 The cash-in-advance model (M1)

2.1 Timing of decisions

The model consists in four types of economic agents (the consumer-household, the good-producing firm, the financial intermediary and the central bank) and five markets (goods, labor, loanable funds, foreign assets and money markets) in a small open economy framework. The timing of decisions within a period can be separated in four steps :

- At the beginning of the period, the monetary shock occurs : the monetary authorities inject liquidity into the loanable funds market.
- The credit market opens. The household uses M_t^b as a bank deposit. The firm determines its demand for labor and capital so as to produce the single worldwide good. As in Dow [1995], the firm borrows cash from the financial intermediary to finance investment.
- In the third step, the perfectly competitive good market opens, production and purchasing decisions are made.
- At the end of the period, the foreign asset market opens. The representative household decides to buy or to sell foreign assets whose return is given by the exogenous foreign interest rate. Labor income is collected and loans are repaid to the financial intermediary. Moreover, as the owner of the firm and the bank, the household receives dividend payments from them.

2.2 Structure of the model

The goods market is perfectly competitive, the domestic firms and the rest of the world compete with each other to produce a single good whose price in domestic currency is P_t. This implies that the law of one price holds, as well as the purchasing power parity. Let e_t be the price of the foreign currency in terms of domestic currency. With P^* the exogenous (small open economy assumption) price of the good in foreign currency units, purchasing power parity is given by the following equation [2]:

$$P_t = e_t P^* \tag{1}$$

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[2] The occurrence of monetary shocks renders the whole set of variables (except the foreign price level P^* and the foreign nominal interest rate i^*) state-contingent. A rigourious notation would thus imply noting variables as $Y(s_t), C(s_t), ...$ with s_t the state of the world contingent to the realized monetary shock. However to ease the reading we abstract from these heavy notations.

Consequently, the real exchange rate, defined as the ratio of the good prices in terms of domestic currency, is equal to one.

2.2.1 The household

Preferences : The representative household maximizes :

$$E_0 = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \tag{2}$$

with C_t the consumption of the good and H_t the worked hours. As the time endowment is normalized to unity, leisure time L_t is given by the following equation:

$$1 = L_t + H_t$$

The utility function is given by :

$$U(C_t, H_t) = \log C_t + \gamma \log(1 - H_t) \tag{3}$$

Constraints : The household faces a cash-in-advance constraint on her consumption purchases :

$$P_t C_t \leq M_t - M_t^b \tag{4}$$

where M_t denotes the entire economy's money stock that is held at the beginning of the period by the household and M_t^b the amount of money holdings put into the bank. The return of the bank deposits (at the end of the period) is given by the nominal interest rate R_t.

The household also saves by holding foreign assets. International financial markets are incomplete and, each period, the household buys B_{t+1} (in foreign currency) of assets issued by the rest of the world. The foreign financial asset yields an exogenous (small open economy assumption) no-risk nominal interest rate i^* tomorrow. As the foreign assets are issued in foreign currency, the nominal exchange rate is a key variable in the portfolio decisions.

In period t, the household decides the amount of domestic holdings M_{t+1} and the amount of foreign assets B_{t+1} that she wants to accumulate. She also determines her consumption of good C_t, her labor supply H_t and the amount of money M_t^b she puts into the bank in the period.

The household budget constraint is then :

$$\begin{aligned} & M_{t+1} + e_t B_{t+1} + P_t C_t \\ \leq \quad & M_t - M_t^b + P_t w_t H_t + (1 + R_t) M_t^b + e_t (1 + i^*) B_t + D_t^f + D_t^b \end{aligned} \tag{5}$$

where w_t denotes the real wage, D_t^f and D_t^b the profits of the firm and those of the bank respectively, which are returned as dividends to the household at the end of the period.

Program : The household maximizes her expected intertemporal utility (2) subject to the cash-in-advance constraint (4) and the budget constraint (5). This program is written as a Bellman equation :

$$V(M_t, B_t) = \underset{\{C_t, H_t, M_t^b, M_{t+1}, B_{t+1}\}}{Max} \{U(C_t, H_t) + \beta E_t V(M_{t+1}, B_{t+1})\}$$

subject to constraints (4) and (5). With λ_t the multiplier associated with the budget constraint, the first order conditions are :

$$\begin{aligned} U'_{Ct} &= (1+R_t)P_t\lambda_t && (6)\\ -U'_{H_t} &= w_t P_t \lambda_t && (7)\\ e_t\lambda_t &= \beta E_t\left[e_{t+1}(1+i^*)\lambda_{t+1}\right] && (8)\\ \lambda_t &= \beta E_t\left[\frac{U'_{Ct+1}}{P_{t+1}}\right] && (9) \end{aligned}$$

$\Lambda_t = P_t\lambda_t$ expresses the shadow price associated with the household real wealth. Equation (6) equates the marginal utility of consumption (U'_{ct}) and the cost of consumption. Consuming one marginal unit of good costs Λ_t in terms of real wealth plus the opportunity cost associated with a bank deposit.

Labor revenues do not enter the cash-in-advance constraint, hence they cannot be used to consume in the current period. Equation (7) states the equality between the cost of working in terms of marginal utility and its benefits in terms of real wealth increase. Equation (8) is related to the choice of foreign assets and equates the current marginal cost (in terms of wealth) of buying foreign assets ($e_t\lambda_t$) to its gains induced the next period ($e_{t+1}\lambda_{t+1}(1+i^*)$). Condition (9) represents the choice of holding domestic money. The household equates the cost of holding money (λ_t) in the current period to the marginal utility of consumption it yields the next period. Holding one unit of domestic currency in the current period allows the consumer to buy $\frac{1}{P_{t+1}}$ units of good the next period, which induces marginal utility U'_{ct+1}.

2.2.2 The firm

The production technology is given by a Cobb-Douglas function :

$$Y_t = K_t^\alpha H_t^{1-\alpha} \tag{10}$$

where $\alpha \in [0, 1]$.

The objective of the representative firm is to maximize the discounted stream of dividends payments where the dividends are discounted by its value to the owner of the firm (the consumer). Consequently, it chooses between paying the dividends to the household at the end of the period and investing them in physical capital. The discounted rate that captures this decision is the ratio of the multipliers associated with the budget constraint of the household, since that ratio reflects the consumer's variation in wealth. Profits from sales are received by the firm at the end of the period. Hence, at the beginning of the period, the firm has to borrow funds from the bank to invest in physical capital. The cost of borrowing is the nominal interest rate R_t, which equates the rate of return on the household bank deposits.

The program of the firm is then :

$$V(K_t) = \underset{\{H_t,K_{t+1}\}}{Max} \left\{ D_t^f + E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] V(K_{t+1}) \right\} \tag{11}$$

with :

$$D_t^f = P_t Y_t - P_t w_t H_t - P_t(1+R_t)I_t \tag{12}$$

given the law of motion of the physical capital stock :

$$I_t = K_{t+1} - (1-\delta)K_t \tag{13}$$

The first order conditions associated with the demand for labor and capital are :

$$w_t = (1-\alpha)\frac{Y_t}{H_t} \tag{14}$$

$$1 + R_t = \beta E_t \left[\frac{P_{t+1}\lambda_{t+1}}{P_t \lambda_t} (\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta + (1-\delta)R_{t+1}) \right] \tag{15}$$

Equation (14) states that the cost of hiring workers equates their marginal productivity. Equation (15) equates the cost of a marginal investment, which incorporates the borrowing cost $(1 + R_t)$, to the benefits of marginal investment. The latter includes the marginal productivity of capital $\alpha\frac{Y_{t+1}}{K_{t+1}}$, the marginal revenues associated with selling the undepreciated capital $(1-\delta)$ as well as the loan that the firm will not have to make thanks to the undepreciated capital $(1-\delta)R_{t+1}$.

2.2.3 The central bank

Each period, an amount of money X_t is injected into the loanable funds market. The money stock evolves according to :

$$M_{t+1} = M_t + X_t \tag{16}$$

with the monetary injection defined as :

$$X_t = (g_t - 1)M_t \tag{17}$$

The money growth factor g_t evolves according to a first order autoregressive process

$$\log g_{t+1} = (1-\rho_g)\log \overline{g} + \rho_g \log g_t + \varepsilon_{gt+1} \tag{18}$$

with ε_{gt+1} a white noise.

2.2.4 The financial intermediary

In the model, the financial intermediary accepts deposits from the household (M_t^b) which are repaid at the end of the period at the interest rate R_t. The bank also receives cash injections X_t from the economy's monetary authorities. The bank's resources are loaned to the firm. The end-of-period profit is redistributed to the household in the form of dividends. The asset balance of the bank leads to :

$$P_t I_t = M_t^b + X_t \tag{19}$$

where $P_t I_t$ are loans made to the firm. At the end of the period, the dividends of the bank are

$$D_t^b = (1 + R_t) P_t I_t - (1 + R_t) M_t^b \tag{20}$$

Using (19), (20) becomes :

$$D_t^b = (1 + R_t) X_t \tag{21}$$

2.3 Equilibrium

The equilibrium is characterized by the set of prices $\Omega_t^P = \{w_t, R_t, P_t, e_t\}_{t=0}^{\infty}$ and the set of quantities[3]

$$\begin{aligned} \Omega_t^C &= \left\{C_t, H_t, B_{t+1}, M_t^b, M_{t+1}\right\}_{t=0}^{\infty} \\ \Omega_t^Q &= \{Y_t, H_t, K_{t+1}\}_{t=0}^{\infty} \end{aligned}$$

such as :

- given the set of prices Ω^P, the vector of exogenous foreign variables $\{i^*, P^*\}$, the set of quantities Ω^C maximizes the expected intertemporal utility of the household subject to equations (4) and (5),
- given the set of prices Ω^P, the vector of exogenous foreign variables $\{i^*, P^*\}$, the set of quantities Ω^Q maximizes the profits of the representative firm subject to equations (10) and (13),
- given the sets of quantities Ω^C and Ω^Q and given the vector of exogenous foreign variables $\{i^*, P^*\}$, the set of prices Ω^P ensures that the labor market, the loanable funds market and the money market are cleared and that the purchasing power parity is satisfied.

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[3] For sake of simplicity, we do not distinguish notations of supply variables from demand variables on all markets.

Since we model a small open economy, output and assets domestic aggregate quantities are equilibrium quantities at the given foreign price level P_t^* and the given nominal foreign interest rate i^*. That is, on the foreign assets market the domestic household can carry out any foreign asset she is willing to hold given the foreign interest rate, being only constrained by her budget constraint. We thus infer from the budget constraint (5) and the market equilibria that the household's foreign asset holdings evolve as:

$$e_t B_{t+1} - e_t(1+i^*)B_t = P_t(Y_t - (C_t + I_t)) \tag{22}$$

This equation reflects the equilibrium of the balance of payments of the home economy. The small country trades with the rest of the world, depending on the levels of the home production and absorption, as shown in equation (22). If domestic production exceeds absorption ($Y_t - (C_t + I_t) > 0$), the trade balance is positive while the capital account is negative : the household sells the production surplus abroad and increases her holding of foreign assets. In contrast, if domestic production cannot satisfy the domestic demand for good, the economy has to import goods from the rest of the world and finance its trade deficit by borrowing from abroad.

The set of first-order conditions, market equilibrium equations as well as the laws of motion for physical capital, domestic money supply, foreign assets and monetary growth factor constitutes a non-linear dynamic system (see appendix A.1). To solve the model we proceed the following way. We first determine the long run equilibrium of the economy (section 2.4). Then, following King, Plosser and Rebelo [1988] the dynamic system is log-linearized around this steady state. Decision rules are determined through Farmer [1993]'s methodology. Technical solving is detailed in appendix A.2.

2.4 Calibration and steady state equilibrium

The period in the model is assumed to be a quarter. The calibration of the parameters $\{\alpha, \beta, \delta, H\}$ is standard. The parameter ν stands for the average of the trade balance to GDP ratio for the G7 countries except the United States [4], on the period 1973:1-1998:3. We use this ratio to determine the long run real debt to GDP ratio as shown below. The long run inflation factor π is based on the average inflation factor on G7 countries between 1973:1 and 1997:4 (OECD sources). Calibration for the structural parameters is summarized in table 1. To determine the persistence coefficient of the monetary shock ρ_g and the standard deviation of the monetary innovations $\sigma_{\varepsilon g}$, we run regressions on the monetary base of G7 countries except the United-States, on the period 1973:1-1998:4. Data comes from OECD's *Main Economic Indicators*. Estimates are reported in table 2. The median value of our estimates is our benchmark calibration for the small open economy monetary process.

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[4] The United-States are excluded from our analysis because of our small open economy assumption.

Table 1: Parameters

α	β	δ	H	ν	π
0.36	0.988	0.025	0.2	0.00061	1.014

Table 2: Monetary shock

Country	ρ_g	$\sigma_{\varepsilon g}$
Canada	0.223	0.00807
France	0.162	0.00753
Germany	0.127	0.00897
Italy	0.339	0.00924
Japan	0.502	0.00663
United-Kingdom	0.017	0.00600
Median	0.19	0.0078

The calibration of the structural parameters allows us to further derive the long run values for aggregate variables. The first step consists in redefining the equations for the system to become stationary (see appendix A.1). We then derive the steady state equilibrium.

Steady state equilibrium The steady state equilibrium represents a situation where the agents' expectations are verified and, absent any trend in the model, real variables are constant.

We consider that the long run inflation factor is equal between countries *i.e.* $\pi = \pi^*$. The steady state monetary growth factor that supports long run inflation is then

$$g = \pi$$

Besides, the (stationary) purchasing power parity equation (32) yields that the long run nominal exchange rate is constant *i.e.* the nominal exchange rate change Δe is equal to 1. Uncovered interest rate parity[5] thus implies that domestic and foreign interest rate are equal in the long run *i.e.* $R = i^*$. Combining equations (34) and (37) further gives the expression for the domestic nominal interest rate R :

$$R = \frac{\pi}{\beta} - 1$$

The first order condition on investment for firms determines the capital marginal productivity, hence the steady state capital/output ratio κ:

$$\kappa \equiv \frac{K}{Y} = \frac{1}{\alpha}\left[\frac{1+R}{\beta} - (1-\delta)(1+R)\right]$$

Then, the equation for the production technology (equation (41)), given our calibration for H, yields to the long run value for domestic output

$$Y = \kappa^{\frac{\alpha}{1-\alpha}} H^{1-\alpha}$$

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[5] See equation 23, expressed in log-deviation from steady state below.

We then obtain the capital stock $K = \kappa Y$. The value for investment is derived through equation (39) : $I = \delta K$.

The balance of payments equilibrium (equation (44)) allows us to determine the long run real debt to GDP ratio. Indeed, consider equation (44) divided by output that gives

$$\frac{b}{Y}\left[1 - \left(\frac{1+i^*}{\pi}\right)\right] = \frac{Y-(C+I)}{Y} = \frac{BC}{Y}$$

where BC stands for domestic trade balance. This equation combined with our calibration for ν determines the real debt to GDP ratio:

$$\frac{b}{Y} = \frac{1}{1-\left(\frac{1+i^*}{\pi}\right)}\,\nu$$

We then derive the household stock of foreign asset in real terms:

$$b = \frac{1}{1-\left(\frac{1+i^*}{\pi}\right)}\nu Y$$

This allows us to get the steady state consumption level through equation (44)

$$C = Y - I - b\left[1 - \frac{1+i^*}{\pi}\right]$$

Combined with the cash-in-advance constraint (equation (33)) and the credit market equilibrium (equation (43)), we obtain the steady state value for real balances:

$$m = C + I$$

Hence, from equation (33), the household bank deposit amounts to

$$m^b = m - \pi C$$

The first-order condition on labor demand gives the wage rate

$$w = (1-\alpha)\frac{Y}{H}$$

and the condition for consumption gives the marginal wealth utility. With the preferences defined as equation (3), we get

$$\Lambda = \frac{1}{C(1+R)}$$

and finally, the value for γ_H is obtained through the condition for leisure:

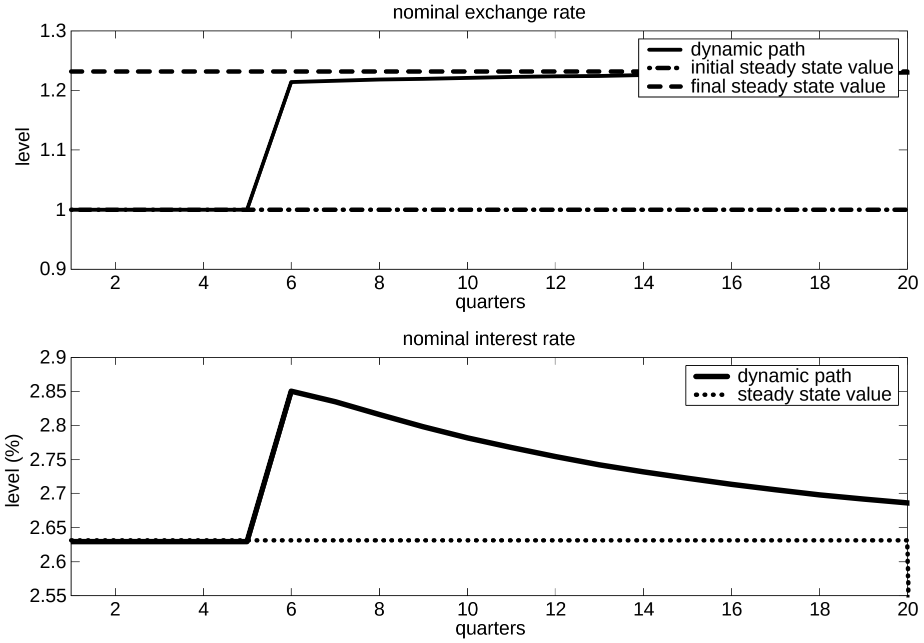
$$\gamma_H = (1-H)w\Lambda$$

2.5 Fisherian effect and the absence of overshooting

Dornbusch [1976] asserts that the nominal exchange rate overshoots its long run value in response to monetary impulses. Therefore, we analyze in this section the aggregate dynamics following an expansionary monetary shock. We then gauge the empirical relevance of the mechanisms of our model by comparing the nominal exchange rate volatility as predicted by the model to that of its monetary fundamentals. As we consider a small open economy, foreign variables (the foreign interest rate and the foreign inflation rate) are constant.

Figure 1 displays the impulse response functions of the nominal exchange rate and the domestic interest rate following a 1% increase in the home money growth factor in period 6.

Figure 1: *Fisher effect and the absence of overshooting*



The monetary injection leads to an increase in the interest rate[6] as well as a continuous depreciation of the nominal exchange rate. Model M1 does not generate any nominal exchange rate overshooting. This failure is due to the combination of the lack of liquidity effect and the uncovered interest rate parity.

Indeed, in model M1, the response of the nominal interest rate obeys its Fisherian fundamentals. In the period of the shock, the household reduces her bank deposits because of expected inflation. The withdrawal of private funds counteracts the money transfer from the government to the financial intermediary. Ultimately the supply of loans shrinks. This implies an increase of the interest rate which raises the cost of investment. The monetary injection does not imply any liquidity effect.

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[6] The steady state value for the nominal interest rate is 2.630% per quarter. On impact the nominal interest rate increases by 22 basis points.

This phenomenon, associated with uncovered interest parity, prevents the model from generating any nominal exchange rate overshooting. For, the assumption of perfect capital mobility yields :

$$\frac{R}{1+R}E_t\widehat{R}_{t+1}-\frac{i^*}{1+i^*}E_t\widehat{i^*_{t+1}}=E_t\widehat{e}_{t+1}-\widehat{e}_t \tag{23}$$

Equation (23) is obtained from linearizing equations (6), (8) and (9). Equation (23) states that, since there are no barriers to international asset trading, the arbitrage between home and foreign assets leads to uncovered interest rate parity. The foreign interest rate being constant, equation (23) becomes :

$$\frac{R}{1+R}E_t\widehat{R}_{t+1}=E_t\widehat{e}_{t+1}-\widehat{e}_t \tag{24}$$

Since the response of the nominal interest rate is determined by its Fisherian fundamentals, the monetary injection does not generate any liquidity effect. The persistent rise in the nominal interest rate implies, through uncovered interest rate parity (24), a continuous depreciation of the nominal exchange rate along the transition path. The model fails to generate any nominal exchange rate overshooting.

The simulation results reported in table 3 confirm the inability of such monetary models to generate plausible exchange rate volatility. Table 3 displays the standard deviation of nominal exchange rates (σ_e), that of the monetary growth factor (σ_g), and the relative nominal exchange rate variability (σ_e/σ_g) : we compute those volatilities for G-7 countries except the United States, since those countries can be considered as small open economies vis-à-vis the United States. They have previously been filtered according to the Hodrick and Prescott [1997]'s method. Data (1973:1-1998:4) come from OECD's *Main Economic Indicator*.

Table 3: Nominal exchange rate behavior vis-a-vis the US Dollar

Country	$\sigma_e(\%)$	$\sigma_g(\%)$	$\frac{\sigma_e}{\sigma_g}$
Canada	2.88	0.82	3.51
France	9.07	0.75	12.05
Germany	8.40	0.90	9.35
Italy	9.05	0.97	9.32
Japan	9.05	0.80	11.35
United-Kingdom	7.37	0.59	12.41
Median	8.73	0.81	10.35
Model M1	1.14	0.78	1.46

Table 3 highlights the strong disconnect between the nominal exchange rate and the monetary growth factor : nominal exchange rate is about 10.35 times more volatile than nominal shocks. Table 3 also reports the exchange rate volatility as predicted by 500 simulations of model M1. The volatility of the nominal exchange rate is hardly higher than that of the monetary impulse, with a relative volatility that equates 1.44. Unlike Dornbusch [1976]'s result, the model fails to generate any amplified movement for the nominal exchange rate following a nominal shock.

The results drive us to explore the role of market rigidities that may amplify the effects of money on the nominal exchange rate. With the interest rate parity, the lasting increase in the nominal interest rate implies a continuous depreciation of the nominal exchange rate. This mechanism accounts for the inability of the model to explain exchange rate volatility. Consequently, we modify the model to generate a persistent liquidity effect.

3 The limited participation model (M2)

3.1 The model

The timing of decisions is modified : we introduce information asymmetries in order to generate a liquidity effect after a money shock. Besides holding money for consumption purchases (M_t^c), the household now uses some amount of money as bank deposits (M_t^b). Information asymmetries are introduced through the limited participation assumption. In the spirit of Fuerst [1992], we now assume that the monetary shock occurs after the household has made her deposit choice.

Furthermore, following Christiano and Eichenbaum [1992]and King and Watson [1996], we introduce adjustment costs on money holdings. Indeed, if the standard limited participation model generates a liquidity effect following a positive monetary shock, the decrease in the interest rate is not strong and persistent enough as compared to the stylized facts. As modeled by Christiano and Eichenbaum [1992], one way to improve the liquidity effect is to modify the environment so that the financial sector remains more "liquid" than the real sector for several periods after the monetary shock. We model this intuition by assuming that adjusting the money-cash M_t^c is costly. If, after the shock the household increases her money-cash by only a small amount, it implies that the withdrawal of her deposits is reduced. Then, in the following period the firm has to absorb a larger share of the economy's funds and the liquidity effect persists over time. In our setting, given uncovered interest rate parity, a large and lasting fall in the interest rate differential implies a significant overshooting of the nominal exchange rate. Thus our model can generate a large exchange rate overshooting which allows us to quantitatively evaluate its role in the exchange rate fluctuations.

3.1.1 The household

We model the previous assumptions in the following way : As in Andolfatto and Gomme [2000], we consider that in the current period the household chooses the amount of deposits she wants to put into the bank the next period. Thus at the end of period t, the household chooses the amount M^c_{t+1} of money available for consumption purchases (money-cash) in period $t+1$ and the amount M^b_{t+1} of money put into the bank (money-deposit) in period $t+1$. Because of adjustment costs on money-holdings, at period t, when the household chooses her amount of money-cash M^c_{t+1} and her complement (the amount of money-deposit M^b_{t+1}), she now takes into account the fact that changing her money holdings M^c_{t+1} is costly: it takes time to reorganize the flow of funds.

We assume that the time spent on reorganizing the flow of funds Ω_t is given by :

$$\Omega_t = \frac{\xi}{2}\left(\frac{M^c_{t+1}}{M^c_t} - g\right)^2 \tag{25}$$

In the long run steady state, $\frac{M^c_{t+1}}{M^c_t}$ is equal to g. Then both the level of Ω_t and its derivative with respect to $\frac{M^c_{t+1}}{M^c_t}$ equate zero in steady state. Changing M^c_t is costly (in terms of time) with a marginal cost being an increasing function of the parameter ξ.

Leisure is defined as :

$$L_t = 1 - H_t - \Omega_t$$

The representative household program is now written as:

$$V(M^c_t, M^b_t, B_t) = \underset{\left\{C_t, L_t, M^c_{t+1}, M^b_{t+1}, B_{t+1}\right\}}{Max} \left\{U(C_t, L_t) + \beta E_t V(M^c_{t+1}, M^b_{t+1}, B_{t+1})\right\} \tag{26}$$

subject to the cash-in-advance constraint (equation (27)) and the budget constraint (equation (28)):

$$P_t C_t \leq M^c_t \tag{27}$$

$$\begin{aligned} & M^c_{t+1} + M^b_{t+1} + e_t B_{t+1} + P_t C_t \\ \leq \quad & M^c_t + P_t w_t(1 - L_t - \Omega_t) + (1+R_t)M^b_t + e_t(1+i^*_t)B_t + D^f_t + D^b_t \end{aligned} \tag{28}$$

The first-order conditions related to the choice of bank deposits M^b_{t+1} and to the choice of money-cash M^c_{t+1} now become:

$$\lambda_t = \beta E_t\left[(1+R_{t+1})\lambda_{t+1}\right] \tag{29}$$

$$\begin{aligned} & P_t w_t \lambda_t \frac{\xi}{M^C_t}\left(\frac{M^c_{t+1}}{M^c_t} - g\right) + \lambda_t \\ = \quad & \beta E_t\left[\frac{U'_{Ct+1}}{P_{t+1}}\right] + \beta E_t\left[P_{t+1} w_{t+1} \lambda_{t+1} \frac{\xi M^c_{t+2}}{(M^c_{t+1})^2}\left(\frac{M^c_{t+2}}{M^c_{t+1}} - g\right)\right] \end{aligned} \tag{30}$$

Equation (29) equates the costs to the benefits of a bank deposit. Placing one unit of money in the bank in the current period costs the shadow price λ_t but yields the expected return $(1+R_{t+1})$ which increases the household's wealth by λ_{t+1}.

Equation (30) equates the costs (the left hand side) to the benefits (the right hand side) related to the choice in period t of the amount of money holdings available for consumption in $t+1$. With $\xi = 0$, *i.e.* without adjustment costs, equation (30) is similar to equation (9). When $\xi \neq 0$, the household compares the cost of changing M_{t+1}^c today (time available to work is reduced) to the advantages such a decision will generate tomorrow: in terms of purchasing power and of time saved. Increasing M_{t+1}^c costs some fraction of time today ($\frac{\partial \Omega_t}{\partial M_{t+1}^C} > 0$) but it also implies saving time tomorrow ($\frac{\partial \Omega_{t+1}}{\partial M_{t+1}^C} < 0$).

3.1.2 The firm, the banks and the government

The behavior of the firm, the bank and that of the monetary authorities is not affected by our assumption of limited participation, hence the equations derived in the subsections 2.2.2, 2.2.4 and 2.2.3 still apply.

3.2 Calibration

To solve the model we proceed the same way as presented in section 2.4. The variable M_t^c is redefined as $m_t^c = M_t^c / P_{t-1}$. We then determine the long run equilibrium of the economy. First, recall that the limited participation assumption stands for informational asymmetries on the credit market that disappear in the steady state, given our definition of the long run equilibrium. Second, adjustment costs on money holdings are null in the long run. Therefore the steady state of model M2 is similar to the one of the model M1 (section 2.4). We only have to define the long run value of the (stationary) money holdings through the money market equilibrium equation:

$$m^c = m - m^b$$

For the comparison between both models to be truly convincing, the calibration of the structural parameters is not modified as compared to the one displayed in section 2.4. We only have to give a value for the parameter ξ that scales the adjustment cost on money holdings. The lack of estimates of the cost function parameter ξ induces us to adopt the following strategy: we evaluate the implications of positive monetary shocks on the behaviors of the interest rate and the nominal exchange rate, for different values of ξ, both in terms of impulse response function (section 3.3) and in quantitative terms (section 3.4). In the same spirit as Christiano and Eichenbaum [1992] and King and Watson [1996], we seek to analyze the consequences of the introduction of very small adjustment costs on money holdings: we thus evaluate the chosen values of ξ as regards to the corresponding cost of steady state leisure and in terms of minutes per week.

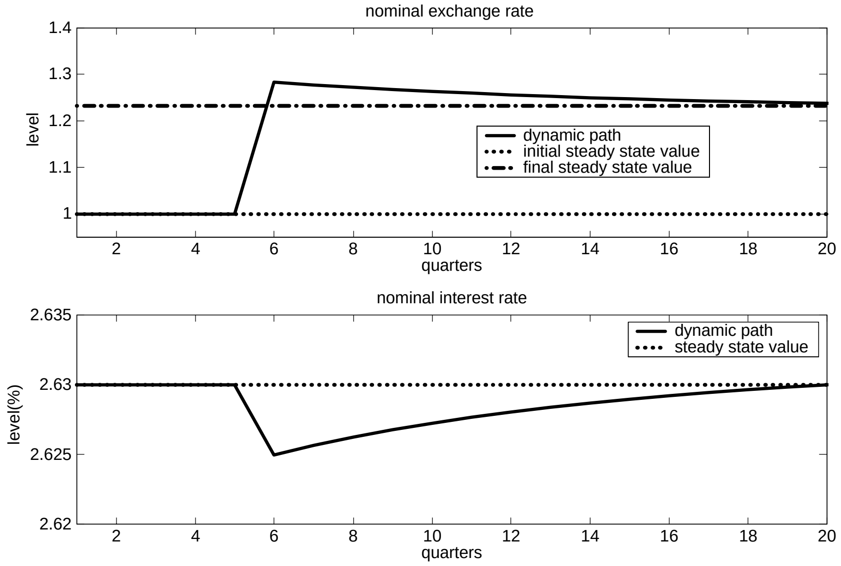
As in model M1, the relevant equations system (see appendix B.1) is then log-linearized around the steady state according to the Farmer [1993]'s methodology. Technical solving is detailed in appendix B.2.

3.3 Liquidity effects and exchange rate overshooting

3.3.1 The simple limited participation model ($\xi = 0$)

In this section we analyze the implications of the simple limited participation assumption, by setting the adjustment costs parameter equal to zero. Figure 2 exhibits the impulse response functions of the nominal exchange rate and the interest rate following a positive shock on the home money growth rate. The model displays a persistent liquidity effect following a positive monetary shock. From that point of view, the transition dynamics is more consistent with the stylized facts (Kim and Roubini [2000]).

Figure 2: *Liquidity effect and nominal exchange rate overshooting* ($\xi = 0$)



The limited participation model with investment as a cash good -and without adjustment costs on money holdings- is able to reproduce a persistent liquidity effect. According to uncovered interest rate parity, an expected fall of the interest rate differential is offset by an expected appreciation of the exchange rate. Consequently, from the second period on, the exchange rate appreciates before reaching its new steady state : our model generates a nominal exchange rate overshooting following a expansionary monetary shock. The specific path of the exchange rate stems from the persistent decrease of the interest rate.

A persistent but weak liquidity effect When money is injected into the financial system, the household cannot withdraw her deposits within the period. In response to this excess in the money supply, the nominal interest rate goes down. This instantaneous liquidity effect entices the firm to invest more. The rise in investment today is all the more important as she anticipates a relative increase in R_{t+1} the nominal interest rate the next period. Indeed, even if the interest rate remains below its steady state level in the following periods, the firm anticipates that the household will reduce her deposits, which implies a relative decrease in the money supply and a rise in the interest rate level compared with its value in the first period.

The immediate liquidity effect persists over the subsequent periods for two reasons : firstly the interest rate decrease stems from the fall in the demand for loans. Indeed, with the instantaneous upsurge in investment, one period after the shock, the economy inherits a large capital stock, which reduces its marginal productivity. The firm is then enticed to cut its investment. As a result, one period after the monetary injection, the decrease in the demand for loans outweighs the reduction in bank deposit. The interest rate remains below its steady state level, the liquidity effect persists over time. However, the second period on, the magnitude of the liquidity effect remains weak[7].

A weak exchange rate overshooting The instantaneous decrease in the nominal interest rate reduces the return on home savings, which entices the representative household to hold more foreign assets. This leads to an instantaneous depreciation which is further followed by a continuous appreciation. Indeed, the subsequent dynamics of the nominal exchange rate is determined by uncovered interest rate parity (24). Since the liquidity effect is expected to be persistent ($E_t\widehat{R}_{t+1} < 0$), the significant and persistent negative interest rate differential is counterbalanced by the expected appreciation of the exchange rate ($E_t\widehat{e}_{t+1} - \widehat{e}_t < 0$). The exchange rate adjustment to its new steady-state is all the slower as the interest rate differential is persistent. Model M2 is indeed able to account for the nominal exchange rate overshooting. Yet the limited magnitude of the liquidity effect after the first period translates into a weak exchange rate overshooting.

3.3.2 Introducing adjustment costs on money holdings

In order to assess the implications of introducing adjustment costs on money holdings, we analyze the dynamics implied by a positive monetary shock firstly when $\xi = 10$. With this calibration, an 1% increase in the ratio $\frac{M^c_{t+1}}{M^c_t}$ beyond its stationary value costs 0.06% of the steady state value of leisure. We also evaluate the dynamics implied by a monetary shock for a value of ξ equal to 30. With this calibration, an 1% increase in the ratio $\frac{M^c_{t+1}}{M^c_t}$ beyond its stationary value costs 0.2% of the steady state value of leisure : we still consider very small adjustment costs. Indeed, when we translate into minutes per week the amount of time lost in rearranging portfolio, we obtain that the household spends around 4 minutes per week when she increases her money holdings 1% beyond the steady-state value for $\xi = 10$. For $\xi = 30$, it represents around 13 minutes per week. We base this calculation on Juster and Stafford [1991]'s estimates of weekly leisure time. The impulse response functions are displayed in figure 3[8].

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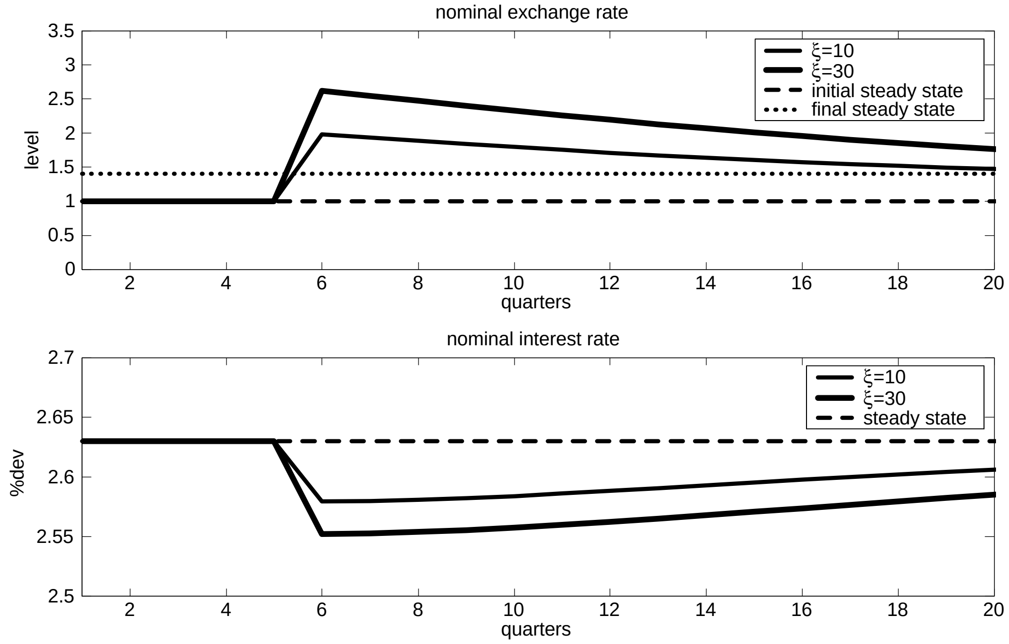
[7] Indeed, in the period of the shock the nominal interest rate equates 2.6252%, while it amounts to 2.63% in the steady state equilibrium. It implies a 0.64 basis point decrease on impact.

[8] For $\xi = 10$, the immediate decrease in the nominal interest rate amounts to 5 basis points, while it equates 8 basis points for $\xi = 30$.

Figure 3: *Liquidity effect, nominal exchange rate and adjustment costs*



If the investment dynamics contributes to the persistent fall in the interest rate, the behavior of the supply for loans plays a crucial role in explaining the magnitude of the liquidity effect. In the period of the monetary shock, the household chooses the amount of money that she wants to consume tomorrow (M^c_{t+1}) and the amount of money that she will put into the bank tomorrow (M^b_{t+1}). After the occurrence of the money shock, the agent anticipates inflation : as in the previous model, the household wants to preserve her consumption in the future by increasing today the amount of nominal money balances. However, it is costly for the household to raise the ratio $\frac{M^C_{t+1}}{M^C_t}$ dramatically. Changing M^c_{t+1} deprives the agent of time available for leisure or labor. According to equation (30), with larger adjustment costs, it is more expensive to modify money holdings today and the household will rather wait. As a result, in the first period, the household raises the amount of money-cash M^c_{t+1} by a small amount. This implies that the decrease in private deposits is reduced in the second period. Hence the second period on, the withdrawal of money-deposits does not counteract the persistent increase in money supply coming from the monetary authorities : This exerts a downward pressure on the nominal interest rate.

Ultimately the increase in loan supply and the decrease in loan demand induce the fall in the interest rate : both mechanisms contribute to the large and persistent liquidity effect and to enlarge the magnitude of overshooting. Given uncovered interest rate parity, overshooting results from the requirement that the interest rate differential is equal to the expected rate of appreciation.

In order to generate a substantial overshooting, our paper as well as Dornbusch [1976]'s model introduce rigidities on the market where the nominal interest rate is determined. Dornbusch [1976] considers price stickiness which magnifies the nominal interest rate response to monetary shocks on the money market. In our setting, the rigidity lies in the credit market with the predetermined bank deposit and the adjustment costs on money holdings. Both models are then able to generate a liquidity effect and a subsequent overshooting dynamics for the nominal exchange rate.

Beyond the qualitative response of the nominal exchange rate following a monetary expansion, our objective is to assess whether, from a quantitative point of view, the overshooting phenomenon explains the nominal exchange rate volatility.

3.4 Overshooting and volatility of the nominal exchange rate

In this section, we measure whether overshooting is responsible for the high volatility of the nominal exchange rate. In order to assess the empirical relevance of overshooting, we compare the nominal exchange rate volatility predicted by the model that fails to generate any overshooting (model M1) with the one that displays overshooting (model M2), for the three chosen values for ξ. Table 4 reports the theoretical results from both models. The statistics are obtained from 500 simulations of the series that are filtered according to the Hodrick and Prescott [1997]'s method.

Table 4: Theoretical and empirical volatilities

	$\sigma_e(\%)$	σ_e/σ_g
M1	1.12	1.42
M2 : $\xi = 0$	1.21	1.52
M2 : $\xi = 10$	1.78	2.24
M2 : $\xi = 30$	2.44	3.10
Data	8.73	10.35

The introduction of adjustment costs substantially improves the predictions of the model concerning the volatility of the nominal exchange rate. Indeed, the theoretical nominal exchange rate is more than three times more volatile than monetary shocks. The results presented in table 4 confirm that overshooting significantly affects the nominal exchange rate variability. Nominal exchange rate fluctuations are disconnected from those of its fundamentals. The higher ξ the more limited the withdrawal of private deposits. Hence, the fall in the interest rate and the depreciation of the exchange rate are larger, which implies more exchange rate volatility. Note that the simple limited participation model (without adjustment costs) is unable to generate a plausible exchange rate volatility : the results highlight the crucial role played by the adjustment costs. Yet, only small adjustment costs on money holdings added to frictions in the credit market *via* the limited participation assumption are needed to generate a plausible exchange rate volatility following a monetary shock. In accordance with Dornbusch [1976], following monetary impulses, overshooting plays a key role in explaining the high volatility of the nominal exchange rate.

Even if both absolute and relative exchange rate volatilities are still too weak compared to the stylized facts, the introduction of the assumption of limited participation in presence of adjustment costs improves the predictions of the model and contributes to a better understanding of the nominal exchange rate behavior. As our aim was to explain only partially the nominal exchange rate volatility via the overshooting dynamics, we think that the results presented in table 4 confirm this intuition. An overshooting dynamics is associated with a higher nominal exchange rate volatility. Furthermore, we compute the first-order autoregressive coefficient for the nominal exchange rate in order to evaluate whether our model correctly captures the high persistence observed in the data. The median autocorrelation equates 0.80 in the industrialized countries of our sample. In the limited participation model with adjustment costs on money holdings, the theoretical autocorrelation equals 0.64 [9]. Our model is able to account for a substantial part of the persistence in the exchange rate movements. We thus obtain the same results as Chari *and al.* [2001] with a much simpler model.

Moreover, the limited participation model in a small-open economy context is consistent with the conditional features of the business cycles following a monetary shock. We have already underlined that a positive monetary shock generates a decrease in the nominal interest rate, a nominal exchange rate depreciation (followed by an appreciation) and an increase in the price level. These impulse response functions are consistent with the empirical implications of monetary innovations, as identified through alternative VAR specifications by Christiano, Eichenbaum and Evans [1997] in a closed-economy context, Kalyvitis and Michaelides [2001], Kim and Roubini [2000] and Schlagenhauf and Wrase [1995] in an multi-country setting. Furthermore, Schlagenhauf and Wrase [1995] and Christiano and Eichenbaum [1992] identified the output impulse response function following monetary expansion for the G7 countries. They show that for all countries output initially decreases for six to nine months, and further increases. Therefore we check whether the models generate output impulse response functions consistent with this finding. Figures 4 and 5 display the output impulse response functions for models M1 and M2.

Figure 4 displays a negative and persistent response for domestic output with a positive domestic monetary shock, which is counterfactual. Because of the cash-in-advance constraint, the household reduces her consumption level given current and expected inflation, to rather report on leisure. Hence labor supply and worked hours contract. In the period of the shock, with the capital stock predetermined output thus falls with employment. As the effects of the monetary shock diminish, output monotonically comes back to its initial steady state level.

Unlike the cash-in-advance model, the limited participation model correctly accounts for output dynamics following a monetary shock, as displayed in figure 5 for $\xi = 0$, $\xi = 10$ and

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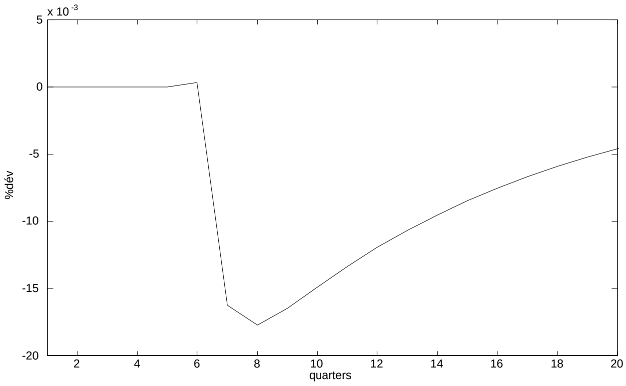
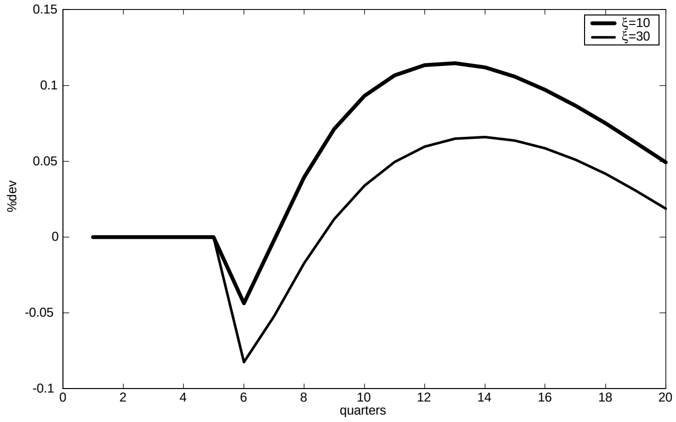
[9] This result is not very sensitive to the value of ξ.

Figure 4: *Output response in the cash-in-advance model (M1)*



$\xi = 30$. Indeed, after an initial decline below its steady state level, output increases following a positive monetary injection.

Figure 5: *Output response in the limited participation model (M2)*



In the limited participation model, the expansionary monetary shock generates a positive wealth effect. Because of the cash-in-advance constraint and adjustment costs on money holdings, the household does not allocate this excess revenue to consumption. She would rather reduce its working time. On impact, labor supply decreases, so does employment. The stock of physical capital being predetermined, the output response mimics the instantaneous fall in labor supply. The second period on, the investment boom raises the stock of physical capital and output increases beyond its initial steady state value. In the simple cash-in-advance model, the wealth effect on labor supply generates a persistent decrease in employment and output which is counterfactual. Comparing both output impulse response functions (figures 4 and 5) allows us to conclude in favor of the limited participation model which is consistent with empirical evidence.

Finally we check that the nominal exchange rate volatility is not induced by a similar interest rate volatility : Indeed, we check that the model generates a variability of the nominal interest rate consistent with data. For $\xi = 30$, the nominal interest rate volatility as predicted by the model equates 0.08%. The observed volatility of the nominal interest rate based on the OECD HP-filtered series (1973:1-1998:4) amounts to 0.40% for the median of the G7 countries except the United States. The variability of the nominal interest rate as predicted by the model is thus too low compared to the stylized facts. However, this result shows that the model generates an excess volatility of the nominal exchange rate without an implausible volatility of the nominal interest rate. Credit market frictions generate a disconnect between exchange rate and monetary fundamentals that is not an artefact.

The limited participation model with adjustment costs on money holdings is thus able to mimic the empirical consequences of a positive monetary shock, namely a persistent liquidity effect and an increase in output. In quantitative terms, the implied volatility of the nominal exchange rate is disconnected from the one of the monetary growth factor. The overshooting dynamics thus accounts for a substantial part of the nominal exchange rate volatility.

3.5 Robustness of the results

This section demonstrates the robustness of our results by implementing a sensitivity analysis to the key assumptions of the model. As shown in the literature (Christiano [1991], Dow [1995]), the global performances of limited participation models are likely to depend namely on the degree of persistence in the monetary process, on the intertemporal elasticity of substitution and from costly capital adjustment. We thus gauge whether our model is robust to alternative specifications regards these assumptions.

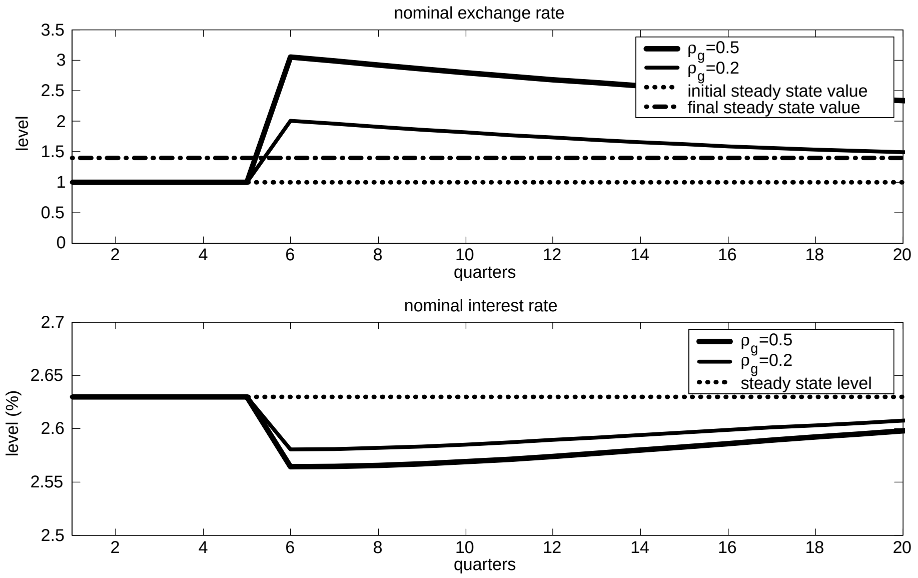
3.5.1 Persistence in the monetary process

In closed economy setting, Christiano [1991] and Christiano and Eichenbaum [1992] underline the fact that the liquidity effect is easily counterbalanced by the Fisher effect as soon as the monetary shock is persistent enough. In our framework, our results could then be biased in the sense that the low persistence of the money growth shock ($\rho_g = 0.19$ in the calibration) could be largely responsible for the exchange rate overshooting.

In order to check the robustness of our results, we conduct a sensitivity analysis to the crucial parameter ρ_g. The figure 6 presents the impulse response functions of the nominal interest rate and the nominal exchange rate for both $\rho_g = 0.2$ and $\rho_g = 0.5$ (such as in Japan).

It shows that the overshooting dynamics is robust to higher monetary growth autocorrelation. The specific path of investment and the limited participation assumption given adjustment costs on money holdings imply that the liquidity effect persists over time, even when $\rho_g = 0.5$.

Figure 6: *Sensitivity analysis to ρ_g*



3.5.2 Intertemporal elasticity of substitution

Section 3.4 makes clear that the output response crucially depends on the wealth effect of the monetary shock and the implied response of worked hours. This drives us to consider the role of the intertemporal elasticity of substitution. The instantaneous utility function (equation (3)) implies a relative risk aversion degree parameter equal to 1 (*i.e.* an intertemporal elasticity of substitution equal to 1 as well). To get a better insight on the role of this parameter, we slightly modify the model to rather consider the following utility function, as in Christiano [1991]:

$$U(C_t, L_t) = \frac{\left[C_t^{1-\gamma} L_t^{\gamma}\right]^{1-\sigma}}{1-\sigma}, \quad 0 < \gamma < 1 \text{ and } \sigma > 0 \tag{31}$$

σ represents the inverse of the intertemporal elasticity of substitution and γ the relative weight of leisure in the above preferences. After solving the limited participation model[10], we determine its quantitative properties by focusing attention on the role for σ.

As expected, the impulse response function analysis underlines that this parameter alters the response of output through the response of worked hours. As shown by Christiano [1991], the parameter σ determines the degree of substitutability between consumption and leisure. For $\sigma > 1$, leisure and consumption are shown to be complementary while for $\sigma < 1$ they are substitutes[11]. As a result, following a positive monetary shock, the immediate decrease

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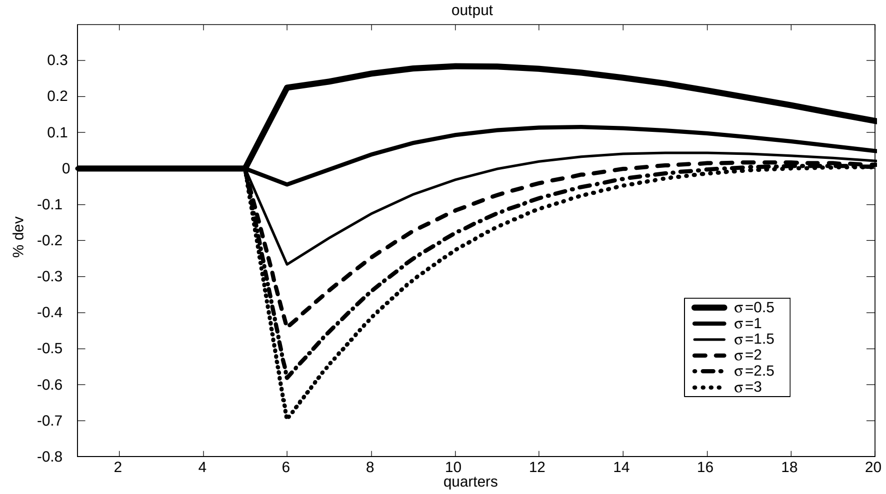
[10] As only the instantaneous utility function has changed, the model mailnly builds on the same structure than the one detailed in appendix. Long run equilibrium and the first order condition relative to consumption and leisure are modified.

[11] This comes from the sign of the cross marginal utility. For $\sigma > 1$, $U'_{LC} < 0$: a decrease in consumption increases the marginal utility of leisure and leisure time raises. For $\sigma < 1$, $U'_{LC} > 0$.

in consumption (Fisher effect) has opposite effects on leisure depending on σ. If $\sigma > 1$, the household would rather increase her leisure time. As a result, labor supply and worked hours decrease. Therefore, the capital stock being predetermined in the period of the shock, output decreases on impact. A nominal shock thus implies a negative response in output. The following period the increase in capital counteracts the reduction in employment and output increases. Figure 7 displays the output impulse response functions according to the values of σ.

Figure 7: *Ouput response and risk aversion*



We further derive the quantitative implications of the sensitivity analysis to σ regards nominal exchange rate volatility. For the calibration displayed in tables 1 and 2, given $\xi = 10$, we obtain the absolute and relative nominal exchange rate volatility presented in table 5.

Table 5: Sensitivity analysis

σ	$\sigma_e(\%)$	$\frac{\sigma_e}{\sigma_g}$
0.5	1.5524	1.9541
1	1.7046	2.1455
1.5	1.7666	2.2237
2	1.7982	2.2634

A higher σ increases the nominal exchange rate volatility. This result stems from adjustments on the credit market. When the monetary shock occurs, the demand for loan increases : firms immediately increase their investment to take advantage of the temporary fall in the nominal interest rate. This shift in the demand for loan tends to dampen the contemporaneous liquidity effect. However, when the household is more willing to smooth her consumption across time (σ high), following monetary injection, as shareholder of the firm, she is reluctant to invest more money into the firm to increase its investment, which limits the instantaneous shift in the demand for loan. The fall in the interest rate, therefore the overshooting dynamics, is enlarged by a intertemporal elasticity of substitution.

To conclude, the influence of the relative risk aversion degree σ is twofold. The more the household is risk adverse, the more nominal exchange rate fluctuates following a monetary shock. This result tends to improve the results of the limited participation model. Yet, increasing σ deteriorates the response in worked hours hence the response in output, which makes the model results somewhat inconsistent with the empirical studies on the effects of monetary shocks. Therefore the role of intertemporal elasticity of substitution has opposite effects regards the performance of the model. This result is not so striking as previous studies put into relief this well-known limit of the limited participation class of models (Dow [1995], Christiano [1991] and Christiano and Eichenbaum [1992]).

3.5.3 Sluggish capital adjustment

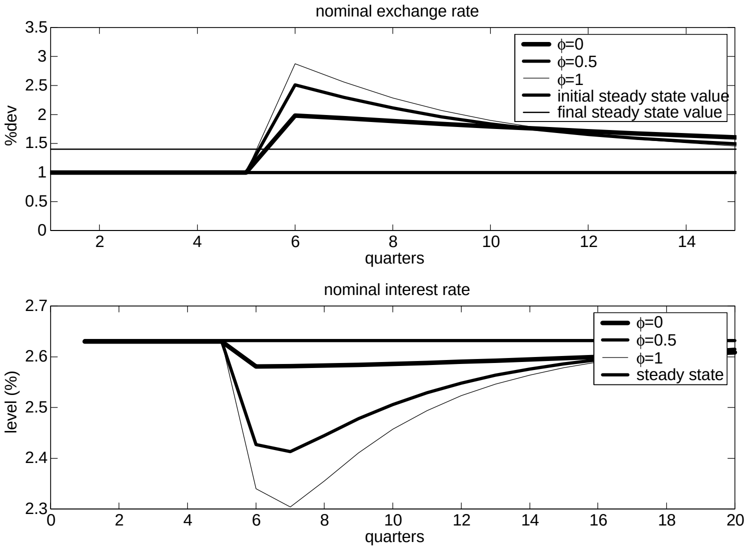
Until now, we have considered that firms are free to adjust physical capital without supporting any cost. However,Dow [1995] and Schlagenhauf and Wrase [1995] insist on the role of sluggish capital adjustment in generating the liquidity effect. Similarly, in model M2, the fall in the interest rate in response to monetary shocks is sensitive to the magnitude of adjustment costs of capital. Indeed, when the monetary shock occurs, firms anticipate that the fall in the nominal interest rate is temporary. As a result, they immediately increase their demand for loans, thus their investment. This shift in the demand for loan tends to reduce the contemporaneous decrease in the interest rate. The instantaneous shift in the demand for loan is lower in the presence of adjustment costs on capital. In order to check this intuition, quadratic adjustment costs on capital $\frac{\varphi}{2}\left(K_{t+1}-K_t\right)^2$ are added to model M2, in the benchmark case of separable preferences (equation 3).

Figure 8 displays the impulse response function for increasing values of φ (for $\xi = 10$). With $\varphi = 1$, following the expansionary monetary policy, the nominal interest rate instantaneously falls by 33 basis points, which is consistent with the magnitude of the liquidity effect found by Dow [1995] and Schlagenhauf and Wrase [1995]. In addition, figure 8 confirms that the higher the adjustment costs on capital, the larger the instantaneous fall in the nominal interest rate following the monetary expansion. Table 6 then confirms that adding adjustment costs on capital enlarges the liquidity effect, thus the exchange rate response to monetary shocks.

Table 6: Sensitivity analysis

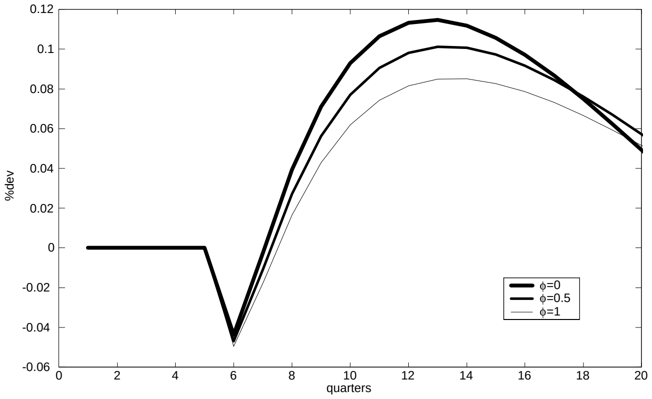
φ	$\sigma_e(\%)$	$\frac{\sigma_e}{\sigma_g}$
0	1.80	2.25
0.5	2.30	2.90
1	2.63	3.30

Figure 8: *Overshooting and adjustment costs on capital*



We finally check that adding adjustment costs on capital does not deteriorate the model results as regards to the output impulse response function.

Figure 9: *Output response and adjustement costs on capital*



Indeed, figure 9 reveals that output still exhibits the *hump shape* dynamics following a positive domestic monetary shock which is consistent with empirical findings.

4 Conclusion

The extremely volatile behavior of the nominal exchange rates since the collapse of the Bretton-Woods system is a well-known stylized fact that a large number of theoretical papers has tried to rationalize. The seminal paper of Dornbusch [1976] highlights the influence of monetary policy on the nominal exchange rate fluctuations. Following a monetary expansion, nominal exchange rate overshoots and this overshooting may explain the large volatility of the exchange rate. Our results demonstrate this proposal in a quantitative general equilibrium model.

This paper puts into relief three salient results. In a model with a cash-in-advance constraint, an expansionary monetary policy implies an increase in the domestic interest rate and a monotonic depreciation of the exchange rate. In a limited participation model where the firm has to borrow to finance its investment, a positive monetary shock is able to generate a persistent decrease in the domestic interest rate and an overshooting of the nominal exchange rate. With small adjustment costs on money holdings, overshooting substantially contributes to the nominal exchange rate volatility. Finally, the overshooting dynamics is magnified in presence of sluggish capital adjustments.

This paper sheds light on the way limited participation affects nominal exchange rate movements. The study of real exchange rate dynamics is left for future research. In addition, one should build an integrating framework featuring pricing-to-market, sticky prices (Chari *and al.* [2001]) and limited participation. Such a model would allow to determine how both mechanisms interact in generating exchange rate fluctuations. Future research following this route might prove fruitful.

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A Technical appendix : The cash-in-advance model (M1)

A.1 Stationarizing the model

As in Hairault and Portier [1993], nominal variables are stationarized by dividing them by the past domestic price level. The nominal exchange rate is redefined as well. We get:

$$m_t = M_t/P_{t-1},\ m_t^b = M_t^b/P_{t-1}, \pi_t = P_t/P_{t-1},\ b_t = e_t B_t/P_{t-1},\ \Delta e_t = e_t/e_{t-1}$$

As foreign assets are expressed in foreign currency, we have to take into account the nominal exchange rate in the expression for b_t. The marginal utility of wealth is given by $\Lambda_t = P_t \lambda_t$. Finally, foreign inflation is defined as $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$.

The relevant equations in the cash-in-advance model are redefined the following way:

$$\begin{aligned}
\pi_t &= \Delta e_t\ \pi_t^* &(32)\\
\pi_t\ C_t &= m_t - m_t^b &(33)\\
U'_{Ct} &= (1+R_t)\Lambda_t &(34)\\
-U'_{Ht} &= w_t \Lambda_t &(35)\\
\Lambda_t &= \beta E_t\left[(1+i_{t+1}^*)\Delta e_{t+1}\frac{\Lambda_{t+1}}{\pi_{t+1}}\right] &(36)\\
\Lambda_t &= \beta E_t\left[\frac{U'_{Ct+1}}{\pi_{t+1}}\right] &(37)\\
1+R_t &= \beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left\{\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta + (1-\delta)R_{t+1}\right\}\right] &(38)\\
I_t &= K_{t+1} - (1-\delta)K_t &(39)\\
w_t &= (1-\alpha)\frac{Y_t}{H_t} &(40)\\
Y_t &= K_t^\alpha H_t^{1-\alpha} &(41)\\
m_{t+1} &= g_t \frac{m_t}{\pi_t} &(42)\\
\pi_t\ I_t &= m_t^b + (g_t - 1)m_t &(43)\\
b_{t+1} - \Delta e_t(1+i_t^*)\frac{b_t}{\pi_t} &= Y_t - C_t - I_t &(44)\\
\log g_{t+1} &= (1-\rho_g)\log \overline{g} + \rho_g \log g_t + \varepsilon_{gt+1} &(45)
\end{aligned}$$

A.2 State-space representation

The space-state system for the cash-in-advance model consists in

- 7 static equations {(32), (33), (34), (35), (40), (41), (43)} corresponding to 7 control variables $\{\pi_t, m_t^b, C_t, H_t, w_t, Y_t, I_t\}$, and
- 7 dynamic equations. The dynamic block splits into 4 backward-looking variables $\{K_t, m_t, b_t, g_t\}$ and 3 forward-looking variables $\{\Delta e_t, \Lambda_t, R_t\}$, which respectively correspond to the set of equations {(39), (42), (44), (45)} and {(36), (37), (38)}.

B Technical appendix : The limited participation model (M2)

B.1 Stationarizing the model

The (stationary) cash-in-advance constraint is now written as:

$$m_t^c = \pi_t C_t \tag{46}$$

and the money market equilibrium requires that

$$m_t = m_t^b + m_t^c \tag{47}$$

The first-order conditions regards the household monetary decisions are modified with the limited participation and adjustment costs on money holdings.

The condition relative to the deposit choice (equation 29) becomes:

$$\Lambda_t = \beta E_t \left[(1 + R_{t+1}) \frac{\Lambda_{t+1}}{\pi_{t+1}} \right] \tag{48}$$

The first-order condition for cash holdings (equation (30)) is written as:

$$w_t \Lambda_t \xi \frac{\pi_t}{m_t^c} (\frac{m_{t+1}^c \pi_t}{m_t^c} - g) + \Lambda_t = \beta E_t \left[\frac{U'_{ct+1}}{\pi_{t+1}} \right] + \beta E_t \left[w_{t+1} \Lambda_{t+1} \xi \frac{m_{t+2}^c \pi_{t+1}}{\left(m_{t+1}^c\right)^2} (\frac{m_{t+2}^c \pi_{t+1}}{m_{t+1}^c} - g) \right] \tag{49}$$

For technical convenience we introduce the following variable:

$$\Delta M_t^c \equiv \frac{M_{t+1}^c}{M_t^c}$$

We redefine this equation in stationarizing the nominal variables according to the previous manner, and we obtain:

$$\Delta M_t^c = \frac{m_{t+1}^c \pi_t}{m_t^c} \tag{50}$$

Thus equation (49) becomes

$$w_t \Lambda_t \xi \frac{\pi_t}{m_t^c}(\Delta M_t^c - g) + \Lambda_t = \beta E_t \left[\frac{U'_{ct+1}}{\pi_{t+1}} \right] + \beta E_t \left[w_{t+1} \Lambda_{t+1} \xi \frac{\Delta M_{t+1}^c}{m_{t+1}^c} (\Delta M_{t+1}^c - g) \right] \tag{51}$$

The relevant equations for solving the limited participation model constitute the following set {(32), (35), (36), (38), (39), (40), (41), (42), (43), (44), (45), (46), (47), (50), (48), (51)}.

B.2 Space-state representation

The space-state system for the cash-in-advance model consists in

- 7 static equations {(32), (47), (46), (35), (40), (41), (43)} corresponding to 7 control variables $\{\pi_t, m_t^b, C_t, H_t, w_t, Y_t, I_t\}$, and
- 9 dynamic equations. The dynamic block splits into 4 backward-looking variables $\{K_t, m_t, b_t, g_t, m_t^c\}$ and 5 forward-looking variables $\{\Delta e_t, R_t, \Lambda_t, \Delta M_t^c, \}$, which respectively correspond to the set of equations {(39), (42), (44), (45), (50)} and {(36), (38), (48), (51)}.