Appendix to

Pricing-to-market, Limited Participation and Exchange Rate Dynamics

A Monopolistic competition, nominal rigidities and mark-up rates

In a monopolistic setting, price-maker firms optimally break the usual equality between sale price and unit cost. Besides, the third price discrimination between countries implies that each firm sets her prices hence two mark-up rates specific to each market, whether local or foreign:

$$p^i_{it}(z) = (1+\mu^i_{it}(z))Cm_{it}(z) \quad (1)$$

$$e_t p^j_{it}(z) = (1+\mu^j_{it}(z))Cm_{it}(z) \qquad j \neq i \quad (2)$$

with $Cm_{it}(z)$ the marginal cost of production of a firm z in country i. To derive the expression for the mark-up rates we first need the total production cost and its derivative, the unit cost (assuming symmetry across firms for notational simplicity). The first step consists in determining the equation of the minimal total production cost for a given amount of production. We then solve the following program:

$$\min_{\{h_{it},k_{it}\}} CT_{it} = P_{it}w_{it}h_{it} + P_{it}z_{it}k_{it}$$

$$s.t \quad x^i_{it} + x^j_{it} = A_{it}k^{\alpha}_{it}h^{1-\alpha}_{it} \geq \overline{F}$$

which yield the following optimal unitary cost:

$$Cm_{it} = P_{it}z^{\alpha}_{it}w^{1-\alpha}_{it}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}$$

The instantaneous profit of a country 1 firm is written as:

$$\pi^f_{1t} = p^1_{1t}x^1_{1t} + e_t p^2_{1t}x^2_{1t} - Cm_{1t}(x^1_{1t} + x^2_{1t}) - P_{1t}i_{1t} - P_{1t}(cp^1_{1t} + cp^2_{1t} + ci_{1t})$$

The first order conditions of the intertemporal program of firm 1 related to the optimal choices for x^1_{1t} and x^2_{1t} are:

$$p^1_{1t} - \nu^1_{1t} - P_{1t}\frac{cp^1_{1t}}{x_{1t}} = Cm_{1t} \quad (3)$$

$$e_t p^2_{1t}(z) - \nu^2_{1t}(z) - P_{1t}\frac{cp^2_{1t}}{x^2_{1t}} = Cm_{1t} \quad (4)$$

Finally, given the definition of the mark-up rates (equations (1) and (2)), we get the optimal expression for the mark-up rate for each domestic and foreign market that the domestic firm can extract:

$$\mu_{1t}^{1} = \frac{\nu_{1t}^{1} + P_{1t}\frac{cp_{1t}^{1}}{x_{1t}^{1}}}{p_{1t}^{1} - \nu_{1t}^{1} - P_{1t}\frac{cp_{1t}^{1}}{x_{1t}^{1}}} \tag{5}$$

$$\mu_{1t}^{2} = \frac{\nu_{1t}^{2} + P_{1t}\frac{cp_{1t}^{2}}{x_{1t}^{2}}}{e_t p_{1t}^{2} - \nu_{1t}^{2} - P_{1t}\frac{cp_{1t}^{2}}{x_{1t}^{2}}} \tag{6}$$

Similar expressions hold for the foreign country.

B Country 2 program

B.1 Country 2 household program, PTM model

Country 2 household program is symmetric to the one described in section 2.1.1 of the paper. The intertemporal optimization problem is written as Bellman equation, expressed in term of domestic currency:

$$V(M_{2t}, B_{2t}(s_t)) = \max\left[U(C_{2t}, L_{2t}) + \beta \int V(M_{2t+1}, B_{2t+1}(s_{t+1}))f(s_{t+1})ds_{t+1}\right]$$

subject to the budget constraint and the cash-in-advance constraints:[1]

$$\begin{aligned} e_t P_{2t} C_{2t} + M_{2t+1} + \int \chi(s_{t+1}) B_2(s_{t+1}) ds_{t+1} &\leq e_t P_{2t} w_{2t} H_{2t} \\ &+ e_t M_{2t} + e_t T_{2t} + B_2(s_t) + \int_n^1 \pi_{2t}^{f}(z)dz \qquad (\lambda_{2t}) \qquad (7) \\ e_t P_{2t} C_{2t} \leq e_t M_{2t} & \qquad (\theta_{2t}) \end{aligned}$$

with λ_{2t} and θ_{2t} the associated Lagrange multipliers. The first order conditions are given by the following equations:

$$U'_{C_{2t}} = e_t P_{2t}\left[\lambda_{2t} + \theta_{2t}\right] \tag{8}$$

$$U'_{L_{2t}} = e_t P_{2t} w_{2t} \lambda_{2t} \tag{9}$$

$$e_t \lambda_{2t} = \beta E_t\left[e_{t+1}\left[\lambda_{2t+1} + \theta_{2t+1}\right]\right] \tag{10}$$

$$\chi(s_{t+1})\lambda_{2t} = \beta \lambda_{2t+1} f(s_{t+1}) \tag{11}$$

Combining the first-order condition for contingents assets made by the country 1 household (equation (9) of the paper) with equation (11) yields to:

$$\int \chi(s_{t+1}) d_{st+1} = \beta \frac{\lambda_{1t+1}}{\lambda_{1t}} f(s_{t+1}) ds_{t+1} = \beta \frac{\lambda_{2t+1}}{\lambda_{2t}} f(s_{t+1}) ds_{t+1}$$

that implies:

$$\beta E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} = \beta E_t \frac{\lambda_{2t+1}}{\lambda_{2t}}$$

This leads to:

$$\lambda_{1t} = \Omega \lambda_{2t}$$

and we set $\Omega = 1$ given our assumption of symmetry across countries.

[1] As presented in next section, foreign firms profits are directly expressed in domestic currency.

B.2 Country 2 firm program, PTM model

The country 2 firm z program is symmetric to the one described in section 2.1.2 of the paper. It is written as a Bellman equation subject to a sequence of instantaneous constraints expressed in domestic currency:

$$V\left[p^1_{2t-1}(z), p^2_{2t-1}(z), k_{2t}(z)\right] = \max \left\{ \begin{array}{c} p^1_{2t}(z)x^1_{2t}(z) + e_t p^2_{2t}(z)x^2_{2t}(z) - e_t P_{2t} w_{2t} h_{2t}(z) - e_t P_{2t} i_{2t}(z) \\ -e_t P_{2t}\left\{cp^1_{2t}(z) + cp^2_{2t}(z) + ci_{2t}(z)\right\} \\ +\int \chi(s_{t+1}) V\left[p^1_{2t}(z), p^2_{2t}(z), k_{2t+1}(z)\right] ds_{t+1} \end{array} \right\}$$

s.t:

$$\begin{array}{rcll} x^1_{2t}(z) + x^2_{2t}(z) & = & A_{2t}k_{2t}(z)^{\alpha}h_{2t}(z)^{1-\alpha} & (\nu_{2t}(z)) \\ x^1_{2t}(z) & \leq & \left[\dfrac{p^1_{2t}(z)}{P^1_{2t}}\right]^{-\eta} D^1_{2t} & (\nu^1_{2t}(z)) \\ x^2_{2t}(z) & \leq & \left[\dfrac{p^2_{2t}(z)}{P^2_{2t}}\right]^{-\eta} D^2_{2t} & (\nu^2_{2t}(z)) \\ k_{2t+1}(z) & = & (1-\delta)k_{2t}(z) + i_{2t}(z) & \end{array}$$

with $\nu^2_t(z)$, $\nu^2_{2t}(z)$ and $\nu^1_{2t}(z)$ the associated Lagrange multipliers. D^1_{2t} and D^2_{2t} are the demand for the foreign aggregate good from both domestic and foreign agents, whose expressions are:

$$\begin{array}{rcl} D^1_{2t} & = & (1-\omega)\left[\dfrac{P^1_{12t}}{P_{1t}}\right]^{-\theta} D_{1t} \\ D^2_{2t} & = & (1-\omega)\left[\dfrac{P^2_{2t}}{P_{2t}}\right]^{-\theta} D_{2t} \end{array}$$

The first-order conditions are the following:[2]

$$\begin{array}{rcl} w_{2t} & = & \dfrac{1}{1+\mu^2_{2t}}\dfrac{p^2_{2t}}{P_{2t}}\left[(1-\alpha)\dfrac{x^1_{2t}+x^2_{2t}}{h_{2t}}\right] \\ q_{2t} & = & \beta E_t\left\{\dfrac{e_{t+1}P_{2t+1}\lambda_{2t+1}}{e_t P_{2t}\lambda_{2t}}\left[q_{2t+1} - \delta + z_{2t+1} + \dfrac{\phi}{2}\left(\dfrac{i_{2t+1}-\delta k_{2t+1}}{k_{2t+1}}\right)^2\right]\right\} \end{array}$$

$$\begin{array}{rcl} p^1_{2t} - e_t P_{2t}\dfrac{cp^1_{2t}}{x^1_{2t}} - \nu^1_{2t} & = & \nu_{2t} \\ e_t p^2_{2t} - e_t P_{2t}\dfrac{cp^2_{2t}}{x^2_{2t}} - \nu^2_{2t} & = & \nu_{2t} \end{array}$$

$$\begin{array}{l} x_{2t} + \beta E_t\left\{\dfrac{\lambda_{2t+1}}{\lambda_{2t}} e_{t+1}P_{2t+1}\Phi\dfrac{p^1_{2t+1}}{[p^1_{2t}]^2} x_{2t+1}\left(\dfrac{p^1_{2t+1}}{p^1_{2t}} - \pi_1\right)\right\} \\ \qquad = x^1_{2t}\eta\dfrac{\nu^1_{2t}}{p^1_{2t}} + \Phi\dfrac{e_t P_{2t} x^1_{1t}}{p^1_{2t-1}}\left\{\dfrac{p^1_{2t}}{p^1_{2t-1}} - \pi_1\right\} \end{array}$$

[2] For notational convenience we directly assume symmetry across foreign firms and suppress the z index.

$$x^2_{2t} + \beta E_t \left\{ \frac{\lambda_{2t+1}}{\lambda_{2t}} e_{t+1} P_{2t+1} \Phi \frac{p^2_{2t+1}}{e_t [p^2_{2t}]^2} x^2_{2t+1} \left(\frac{p^2_{2t+1}}{p^2_{2t}} - \pi_2 \right) \right\}$$
$$= x^2_{2t} \eta \frac{\nu^2_{2t}}{e_t p^2_{2t}} + \Phi \frac{P_{2t} x^2_{2t}}{p^2_{2t-1}} \left\{ \frac{p^2_{2t}}{p^2_{2t-1}} - \pi_2 \right\}$$

with:

$$q_{2t} = 1 + \phi \frac{i_{2t} - \delta k_{2t}}{k_{2t}}$$
$$z_{2t} = \frac{1}{1 + \mu^2_{2t}} \frac{p^2_{2t}(z)}{P_{2t}} \alpha \frac{x^1_{2t} + x^2_{2t}}{k_{2t}}$$

and μ^2_{2t} defined similarly as in appendix A.

B.3 The programs of foreign agents, PTM + LP model

B.3.1 Foreign household

As for the domestic household, the intertemporal program of the foreign household is altered by the introduction of credit market frictions. The corresponding Bellman equation is now:

$$V(M^c_{2t}, M^b_{2t}, B_{2t}(s_t)) = \max \left[U(C_{2t}, L_{2t}) + \beta \int V(M^c_{2t+1}, M^b_{2t+1}, B_{2t+1}(s_{t+1})) f(s_{t+1}) ds_{t+1} \right]$$

subject to the following budget constraint and cash-in-advance constraint:

$$e_t P_{2t} C_{2t} \leq e_t M_{2t} \qquad (\theta_{2t})$$
$$e_t P_{2t} C_{2t} + M^c_{2t+1} + M^b_{2t+1} + \int \chi(s_{t+1}) B_2(s_{t+1}) ds_{t+1} \leq e_t P_{2t} w_{2t} (1 - L_{2t} - \Omega_{2t})$$
$$+ e_t M^c_{2t} + e_t R_{2t} M^b_{2t} + B_2(s_t) + \int_n^1 \pi^f_{2t}(z) dz + e_t \pi^b_{2t} \qquad (\lambda_{2t})$$

The first-order conditions relative to consumption, leisure and contingent assets in the PTM model (equations (8), (9), (11)) still hold. The first-order conditions relative to M^c_{2t+1} and M^b_{2t+1} are the following:

$$e_t \lambda_{2t} = \beta E_t \left[e_{t+1} \lambda_{2t+1} R_{2t+1} \right] \qquad (12)$$

$$e_t \lambda_{2t} + e_t P_{2t} \lambda_{2t} \xi \frac{1}{M^c_{2t}} \left[\frac{M^c_{2t+1}}{M^c_{2t}} - g \right] = \beta E_t \left[\frac{U'_{C2t+1}}{P_{2t+1}} \right]$$
$$+ \beta E_t \left[P_{2t+1} \lambda_{2t+1} e_{t+1} w_{2t+1} \right] \xi \frac{M^c_{2t+1}}{(M^c_{2t+1})^2} \left[\frac{M^c_{2t+2}}{M^c_{2t+1}} - g \right] \qquad (13)$$

B.3.2 Foreign firms

Foreign firms program is slightly changed since investment is now a credit good. The instantaneous profit expression for the foreign firm is now (suppressing the z index):

$$\pi^f_{2t} = p^1_{2t} x^1_{2t} + e_t p^2_{2t} x^2_{2t} - e_t P_{2t} w_{2t} h_{2t} - e_t P_{2t} R_{2t} i_{2t} - e_t P_{2t} \left\{ c p^1_{2t} + c p^2_{2t} + c i_{2t} \right\}$$

Only the first-order condition relative to the capital accumulation optimal decisions has changed as compared to section B.2:

$$q_{2t} + R_{2t} = \beta E_t \left\{ \frac{e_{t+1} P_{2t+1} \lambda_{2t+1}}{e_t P_{2t} \lambda_{2t}} \left[q_{2t+1} - 1 + (1 - \delta) R_{2t+1} + z_{2t+1} + \frac{\phi}{2} \left(\frac{i_{2t+1} - \delta k_{2t+1}}{k_{2t+1}} \right)^2 \right] \right\}$$

C Technical solving of the model

C.1 Stationarizing the PTM model

As in Hairault and Portier [1993], the money stocks and the consumer price indices are stationarized by dividing them by the past (local) consumer price level. Individual prices for firms are expressed in relative prices (divided by the local CPI). The nominal exchange rate and the different constraint multipliers are redefined as well:

$\pi_{it} = \frac{P_{it}}{P_{it-1}}$ $m_{it} = \frac{M_{it}}{P_{it-1}}$ $\Delta e_t = \frac{e_t}{e_{t-1}}$ $\Lambda_{1t} = P_{1t}\lambda_{1t}$ $\Lambda_{2t} = e_t P_{2t}\lambda_{2t}$

$\gamma^1_{1t} = \frac{p^1_{1t}}{P_{1t}}$ $\gamma^2_{1t} = \frac{e_t p^2_{1t}}{P_{1t}}$ $\gamma^1_{2t} = \frac{p^1_{2t}}{e_t P_{2t}}$ $\gamma^2_{2t} = \frac{p^2_{2t}}{P_{2t}}$ $\widetilde{\nu}_{1t} = \frac{\nu_{1t}}{P_{1t}}$

$\widetilde{\nu}_{2t} = \frac{\nu_{2t}}{e_t P_{2t}}$ $\widetilde{\nu}^1_{1t} = \frac{\nu^1_{1t}}{P_{1t}}$ $\widetilde{\nu}^2_{1t} = \frac{\nu^2_{t}}{P_{1t}}$ $\widetilde{\nu}^1_{2t} = \frac{\nu^1_{2t}}{e_t P_{2t}}$ $\widetilde{\nu}^2_{2t} = \frac{\nu^2_{2t}}{e_t P_{2t}}$

The relevant equations in the pricing-to-market model are redefined the following way, with $i = 1, 2$ and $j \neq i$:

$$D_{it} = C_{it} + I_{it} + CI_{it} + CP^i_{it} + CP^j_{it} \quad (14)$$

$$q_{it} = 1 + \phi \frac{I_{it} - \delta K_{it}}{K_{it}} \quad (15)$$

$$\frac{\gamma_H}{1 - H_{it}} = w_{it}\Lambda_{it} \quad (16)$$

$$Y_{it} = A_{it}\ {K_{it}}^{\alpha} {H_{it}}^{1-\alpha} \quad (17)$$

$$Y_{1t} = n(x^1_{1t} + x^2_{1t}) \quad (18)$$

$$Y_{2t} = (1-n)(x^1_{2t} + x^2_{2t}) \quad (19)$$

$$x^1_{1t} = [\gamma^1_{1t}]^{-\theta} \frac{\omega}{D_{1t}} n \quad (20)$$

$$x^2_{1t} = \left[\gamma^2_{1t}\Gamma_t^{-1}\right]^{-\eta} \omega \frac{D_{2t}}{n} \quad (21)$$

$$x^1_{2t} = \left[\gamma^1_{2t}\Gamma_t\right]^{-\eta} (1-\omega) \frac{D_{1t}}{1-n} \quad (22)$$

$$x^2_{2t} = [\gamma^2_{2t}]^{-\eta}(1-\omega)\frac{D_{2t}}{1-n} \quad (23)$$

$$(24)$$

$$
\begin{aligned}
w_{1t} &= \widetilde{\nu}_{1t}\left[(1-\alpha)\frac{Y_{1t}}{H_{1t}}\right] && (25)\\
w_{2t} &= \widetilde{\nu}_{2t}\left[(1-\alpha)\frac{Y_{2t}}{H_{2t}}\right] && (26)\\
z_{1t} &= \alpha\widetilde{\nu}_{1t}\frac{Y_{1t}}{K_{1t}} && (27)\\
z_{2t} &= \alpha\widetilde{\nu}_{2t}\frac{Y_{2t}}{K_{2t}} && (28)\\
\widetilde{\nu}_{1t} &= \gamma_{1t}^{1}-\frac{\Phi}{2}\left[\frac{\gamma_{1t}^{1}\pi_{1t}}{\gamma_{1t-1}^{1}}-\pi_{1}\right]^{2}-\widetilde{\nu}_{1t}^{1} && (29)\\
\widetilde{\nu}_{1t} &= \gamma_{1t}^{2}-\frac{\Phi}{2}\left[\frac{\gamma_{1t}^{2}\pi_{1t}}{\gamma_{1t-1}^{2}\Delta e_{t}}-\pi_{2}\right]^{2}-\widetilde{\nu}_{1t}^{2} && (30)\\
\widetilde{\nu}_{2t} &= \gamma_{2t}^{1}-\frac{\Phi}{2}\left[\frac{\gamma_{2t}^{1}\pi_{2t}\Delta e_{t}}{\gamma_{2t-1}^{1}}-\pi_{1}\right]^{2}-\widetilde{\nu}_{2t}^{1} && (31)\\
\widetilde{\nu}_{2t} &= \gamma_{2t}^{2}-\frac{\Phi}{2}\left[\frac{\gamma_{2t}^{2}\pi_{2t}}{\gamma_{2t-1}^{2}}-\pi_{2}\right]^{2}-\widetilde{\nu}_{2t}^{2} && (32)
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_{t} &= \Gamma_{t-1}de_{t}\frac{\pi_{1t}}{\pi_{2t}} && (33)\\
\Lambda_{2t} &= \Gamma_{t}\Lambda_{1t} && (34)\\
\omega[\gamma_{1t}^{1}]^{1-\theta}+(1-\omega)\left[\gamma_{2t}^{1}\Gamma_{t}\right]^{1-\theta} &= \omega\left[\frac{\gamma_{1t}^{2}}{\Gamma_{t}}\right]^{1-\theta}+(1-\omega)\left(\gamma_{2t}^{2}\right)^{1-\theta} && (35)\\
D_{1t}+\Gamma_{t}D_{2t} &= n(\gamma_{1t}^{1}x_{1t}^{1}+\gamma_{1t}^{2}x_{1t}^{2})+(1-n)\Gamma_{t}(\gamma_{2t}^{1}x_{2t}^{1}+\gamma_{2t}^{2}x_{2t}^{2}) && (36)\\
\pi_{it}C_{it} &= m_{it} && (37)
\end{aligned}
$$

$$
\begin{aligned}
K_{it+1} &= (1-\delta)K_{it}+I_{it} && (38)\\
m_{it+1} &= g_{it}\frac{m_{it}}{\pi_{it}} && (39)\\
\log a_{it+1} &= \rho_{a}\log a_{it}+\rho_{a12}\log a_{jt}+\varepsilon_{ai,t+1}+\psi_{a}\varepsilon_{aj,t+1} && (40)\\
\log g_{it+1} &= \rho_{g}\log g_{it}+\rho_{g12}\log g_{jt}+\varepsilon_{gi,t+1}+\psi_{g}\varepsilon_{gj,t+1} && (41)
\end{aligned}
$$

$$
q_{it}=\beta E_{t}\left[\frac{\Lambda_{it+1}}{\Lambda_{it}}\left\{q_{it+1}-\delta+z_{it+1}+\frac{\phi}{2}\left(\frac{I_{it+1}-\delta K_{it+1}}{K_{it+1}}\right)^{2}\right\}\right] \qquad (42)
$$

$$
\begin{aligned}
&\beta E_{t}\left[\frac{\Lambda_{1t+1}}{\Lambda_{1t}}\Phi\frac{\gamma_{1t+1}^{1}\pi_{1t+1}}{[\gamma_{1t}^{1}]^{2}}x_{1t+1}^{1}\left(\frac{\gamma_{1t+1}^{1}\pi_{1t+1}}{\gamma_{1t}^{1}}-\pi_{1}\right)\right]+x_{1t}^{1}\\
&\qquad=\eta\frac{\widetilde{\nu}_{1t}^{1}}{\gamma_{1t}^{1}}x_{1t}^{1}+\Phi\frac{\pi_{1t}x_{1t}^{1}}{\gamma_{1t-1}^{1}}(\frac{\gamma_{1t}^{1}\pi_{1t}}{\gamma_{1t-1}^{1}}-\pi_{1}) \qquad (43)
\end{aligned}
$$

$$
\begin{aligned}
&x_{1t}^{2}+\beta E_{t}\left[\frac{\Lambda_{1t+1}}{\Lambda_{1t}}\Phi\frac{\gamma_{1t+1}^{2}\pi_{1t+1}}{\Delta e_{t+1}[\gamma_{1t}^{2}]^{2}}x_{1t+1}^{2}\left(\frac{\gamma_{1t+1}^{2}\pi_{1t+1}}{\Delta e_{t+1}\gamma_{1t}^{2}}-\pi_{2}\right)\right]\\
&\qquad=\eta\frac{\widetilde{\nu}_{1t}^{2}}{\gamma_{1t}^{2}}x_{1t}^{2}+\Phi\frac{\pi_{1t}x_{1t}^{2}}{\Delta e_{t}\gamma_{1t-1}^{2}}(\frac{\gamma_{1t}^{2}\pi_{1t}}{\Delta e_{t}\gamma_{1t-1}^{2}}-\pi_{2}) \qquad (44)
\end{aligned}
$$

$$x_{2t}^1 + \beta E_t \left[\frac{\Lambda_{2t+1}}{\Lambda_{2t}} \Phi \frac{\gamma_{2t+1}^1 \pi_{2t+1}}{[\gamma_{2t}^1]^2} \Delta e_{t+1} x_{2t+1}^1 \left(\Delta e_{t+1} \frac{\gamma_{2t+1}^1 \pi_{2t+1}}{\gamma_{2t}^1} - \pi_1 \right) \right]$$
$$= \eta \frac{\widetilde{\nu}_{2t}^1}{\gamma_{2t}^1} x_{2t}^1 + \Phi \Delta e_t \frac{\pi_{2t} x_{2t}^1}{\gamma_{2t-1}^1} (\Delta e_t \frac{\gamma_{2t}^1 \pi_{2t}}{\gamma_{2t-1}} - \pi_1) \tag{45}$$

$$x_{2t}^2 + \beta E_t \left[\frac{\Lambda_{2t+1}}{\Lambda_{2t}} \Phi \frac{\gamma_{2t+1}^2 \pi_{2t+1}}{[\gamma_{2t}^2]^2} x_{2t+1}^2 \left(\frac{\gamma_{2t+1}^2 \pi_{2t+1}}{\gamma_{2t}^2} - \pi_2 \right) \right]$$
$$= \eta \frac{\widetilde{\nu}_{2t}^2}{\gamma_{2t}^2} x_{2t}^2 + \Phi \frac{\pi_{2t} x_{2t}^2}{\gamma_{2t-1}^2} (\frac{\gamma_{2t}^2 \pi_{2t}}{\gamma_{2t-1}^2} - \pi_2) \tag{46}$$

$$\Lambda_{it} = \beta E_t \left[\frac{C_{it+1}^{-\sigma}}{\pi_{it+1}} \right] \tag{47}$$

The set of 44 equations (to (14) to (47)) is associated to a set of 44 variables, for $i = 1, 2$ and $j \neq i$:

- 4 backward-looking variables K_i, m_i,
- 4 exogenous shocks A_i, g_i,
- 36 forward-looking variables $\{C_i, I_i, D_i, Y_i, x_i^i, x_i^j, H_i, w_i, z_i, \gamma_i^i, \gamma_i^j, \Gamma, \Delta e, \Lambda_i, \pi_i, \widetilde{\nu}_i, \widetilde{\nu}_i^i, \widetilde{\nu}_i^j, q_i\}$

C.2 The steady state equilibrium of the PTM model

The steady state equilibrium represents a situation where the agents$'$expectations are verified and, absent any trend in the model, real variables are constant. In the symmetric equilibrium, inflation factors are identical between countries: $\pi_1 = \pi_2 = \pi$. We assume that all relative prices are equal to one: $\Gamma = \gamma_i^i = \gamma_i^j = 1, \forall i = 1, 2$ and $j \neq i$.

The real exchange rate expression (equation (33)) yields that the long run nominal exchange rate is constant i.e. the nominal exchange rate change Δe is equal to 1. Based on equations (39), the steady state monetary growth rate that supports long run inflation is $g_i = \pi, \forall i = 1, 2$.

From the first order condition on prices for firms, and given the relation between μ and η, it comes:

$$\widetilde{\nu}_i^i = \widetilde{\nu}_i^j = \frac{1}{\eta}$$

In the long run adjustment costs on prices are null such equations (29) to (32) yield the following steady state relations :

$$\begin{aligned} \gamma_1^1 - \widetilde{\nu}_1^1 &= \gamma_1^2 - \widetilde{\nu}_1^2 = \widetilde{\nu}_1 \\ \gamma_2^1 - \widetilde{\nu}_2^1 &= \gamma_2^2 - \widetilde{\nu}_2^2 = \widetilde{\nu}_2 \end{aligned}$$

Give this, the first order condition on investment for firms determines the steady state value for z

$$z = \frac{1}{\beta} - (1 - \delta)$$

and the resulting value for the capital/output ratio $\kappa \equiv \frac{K}{Y}$:

$$\kappa = \frac{\alpha \frac{\eta - 1}{\eta}}{z}$$

Given the calibration for H and A, the production function expressed in terms of κ determines the long run value for individual output and on an aggregate level in each country :

$$Y_i = Y = \kappa^{\frac{\alpha}{1-\alpha}} \ H, \forall i = 1, 2$$

The aggregate capital stock identical in each country, is therefore equal to

$$K = \kappa Y$$

From the law of motion for capital (equation (38), $i = 1, 2$), we determine the individual and aggregate investment flow in each country :

$$I = \delta K$$

The assumption that in the long run trade balance is on equilibrium imposes that $Y_i = D_i \ \forall i = 1, 2$, absorption being equal to local production, or the other way round that domestic imports (the left member of equation (48)) equate domestic exports (the right member), that is :

$$\gamma_2^1 x_2^1 = \gamma_1^2 x_1^2 \tag{48}$$

implying that

$$x_i^i = x_i^j = D, \qquad \forall i = 1, 2, j \neq i$$

The definition for aggregate demand therefore delivers the long run value for consumption: $C = D - I$. The real wage derives from the optimal labor demand for firms (identical between countries):

$$w = (1 - \alpha)(1 - \frac{1}{\eta})\frac{Y}{H}$$

The first order condition on consumption yields the long run value for the marginal utility of wealth

$$\Lambda = \beta \frac{C^{-\sigma}}{\pi}$$

The first order condition on leisure yields $\gamma_H = w\Lambda(1 - H)$. Finally, the definition for the nominal interest rate gives the long run nominal interest rate:

$$R = \frac{\pi}{\beta}$$

After the stationarizing of the equations and the determination of the long-run equilibrium, the relevant system of equations is log-linearized around the steady-state equilibrium, according to Farmer [1993]'s methodology. The space-state linearized system is then solved by Dynare for Matlab 7.0.

C.3 Stationarizing the PTM+LP model and solving the steady-state equilibrium

The variables m_{it}^{c} and m_{it}^{b} are respectively defined as $m_{it}^{c} = \frac{M_{it}^{c}}{P_{it-1}}$ and $m_{it}^{b} = \frac{M_{it}^{b}}{P_{it-1}}$. The limited participation assumption stands for informational asymmetries on the credit market that disappear in the long run equilibrium. Besides, adjustment costs on money holdings are null in the steady state equilibrium. As a result, the long run equilibrium of the model is quite similar to the one of the PTM model. From the first order condition for capital accumulation we get the new value for $\kappa \equiv K/Y$:

$$\kappa = \frac{\alpha \frac{\eta-1}{\eta}}{\frac{R}{\beta} - R(1-\delta)}$$

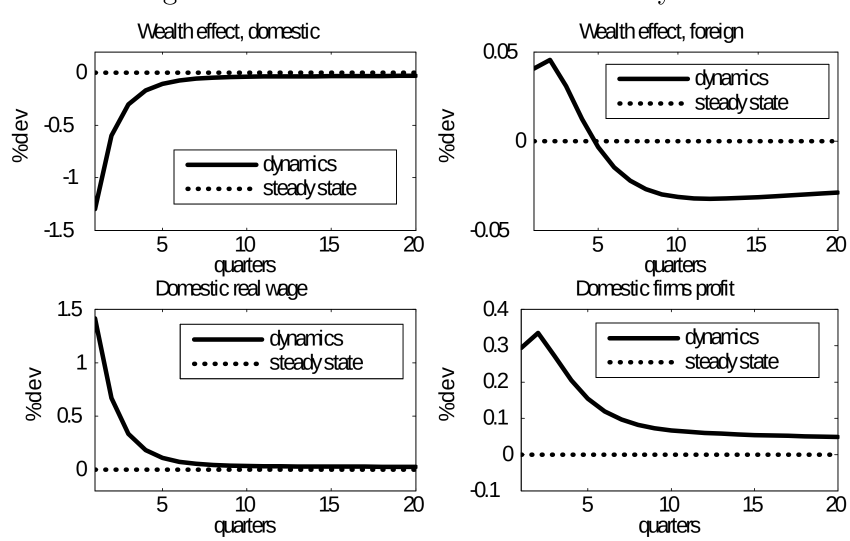
with the steady-state value for R given by the first-order condition on deposits: $R = \frac{\pi}{\beta}$. Regards monetary variables, the cash-in-advance constraint determines the long run value for money-cash: $m^{c} = \pi C$. From the money market equilibrium and the loanable funds market equilibrium (equations (51) and (52) of the paper, stationarized) we get the steady state values for the money stock and money deposits: $m = C + I$ and $m^{b} = m - m^{c}$.

D Complements on the performances of the PTM+LP model

D.1 The benchmark model

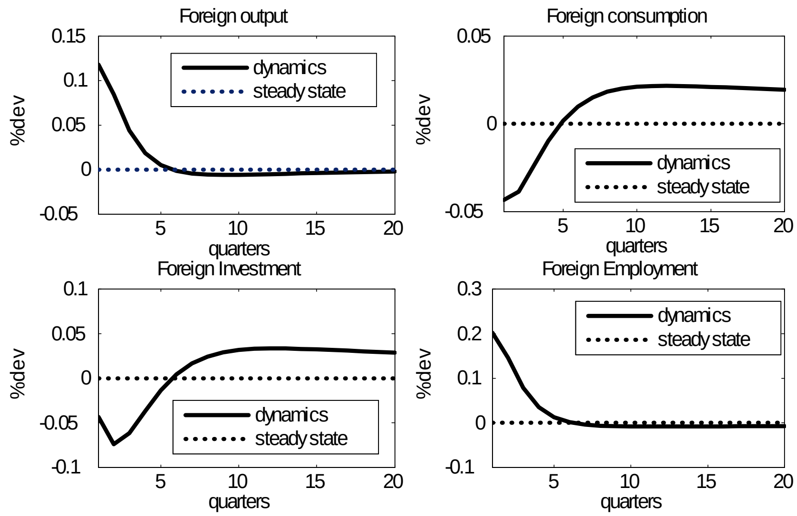
Figures 1 and 2 display the impulse response functions for a large set of domestic and foreign variables, following a 1% increase in the domestic monetary growth rate in period 1.

Figure 1: The effects of a domestic money shock



In the foreign country, the negative wealth effect (figure 2) entices the foreign household to reduce consumption and leisure. As well, investment contracts as the household is the auctioneer of the firms. As a result, foreign aggregate demand decreases below its steady state level. On

Figure 2: Domestic money shock, effects in the foreign country



the contrary, foreign output increases, generating a positive trade balance. Indeed, the domestic investment boom translates into a higher demand, for both domestic and foreign goods. The rise in domestic production is insufficient enough to answer the raise in demand, requiring net imports from the foreign country. Furthermore, foreign firms all the more benefit from the domestic monetary shock as pricing-to-market makes them immune from the real exchange rate depreciation that otherwise would tend to favor domestic goods.

D.2 Sensitivity analysis

D.2.1 Absent any portfolio adjustment costs ($\xi = 0$)

Figure 3 presents the impulse response functions of the nominal and real exchange rates and both domestic and foreign interest rates when $\xi = 0$.

D.2.2 Sensitivity analysis in quantitative terms

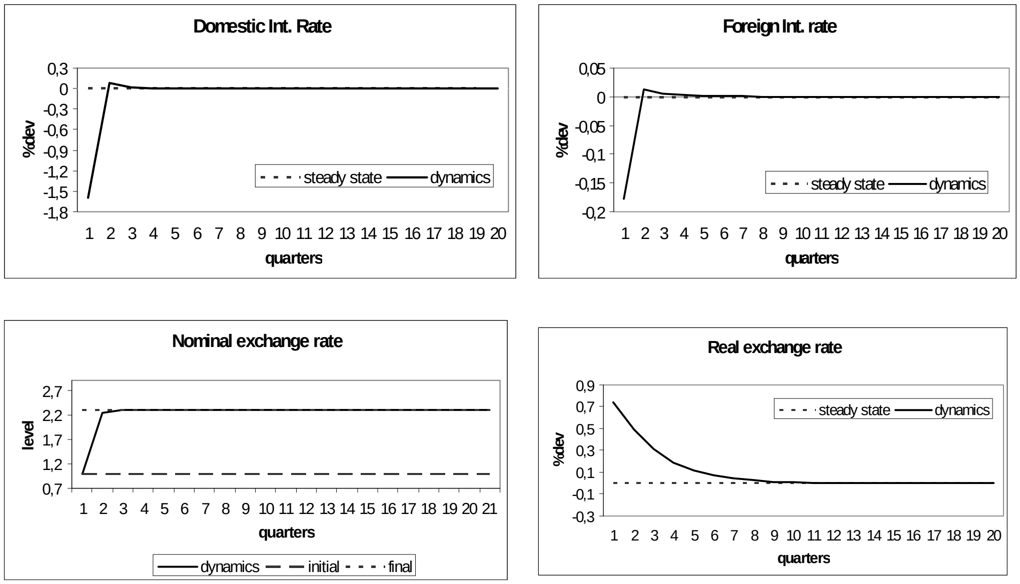
We derive the standard-deviations of nominal and real exchange rates and output for increasing values of Φ, ϕ and ξ when the model is subject to monetary and technological shocks in table 1.

Table 1: Sensitivity analysis (1)

	Credit market frictions ξ				Price rigidities Φ				Capital adj. costs ϕ			
St.Dev. (in %)	0	1	30	50	0	5	30	50	5	10	30	50
Δe	1.43	2.18	3.89	**4.09**	2.22	2.20	2.17	**2.18**	1.30	1.59	2.31	**2.72**
Γ	1.00	1.49	2.84	**2.99**	0	0.93	1.64	**1.82**	0.88	1.10	1.57	**1.83**
Y	0.86	0.83	0.91	**0.91**	1.05	0.93	0.82	**0.82**	0.90	0.88	0.82	**0.90**

Besides, according to Kollmann [2001] the elasticity of substitution between varieties θ has a

Figure 3: In the absence of portfolio adjustment costs



significant influence on international comovements of output, namely in response to technological shocks. We investigate this point through a sensitivity analysis to θ. Table 2 displays the standard-deviation (in %) of Δe, Γ and Y and the cross-country output correlation ($\rho(Y_1, Y_2)$) when the model is subject to technology shocks (columns 2-5) and to both shocks (columns 6-9).

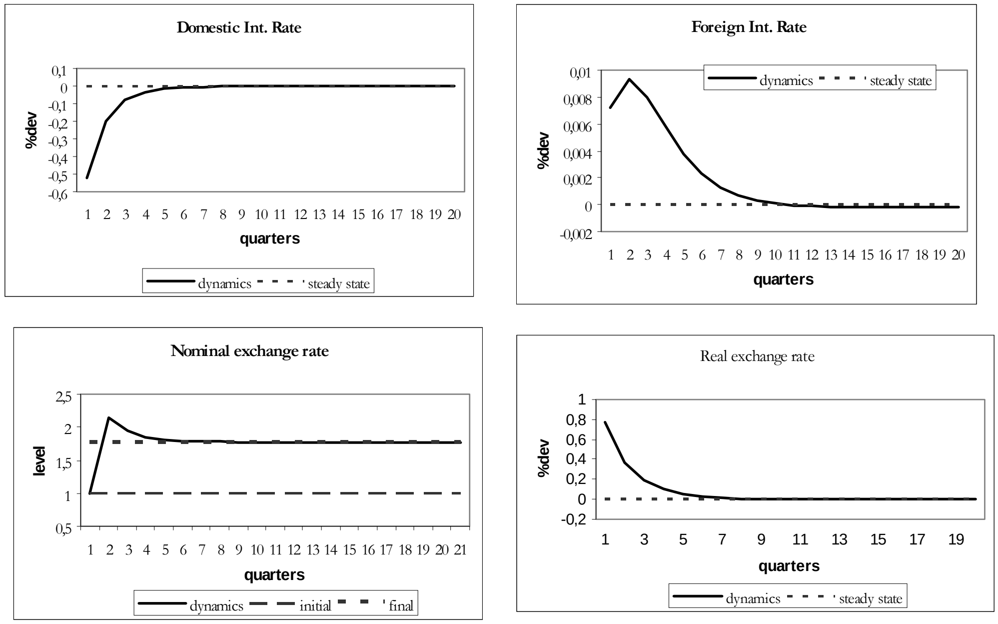
Table 2: Sensitivity analysis (2)

	Techn. Shocks				Both Shocks			
θ	$\sigma(\Delta e)$	$\sigma(\Gamma)$	$\sigma(Y)$	$\rho(Y_1, Y_2)$	$\sigma(\Delta e)$	$\sigma(\Gamma)$	$\sigma(Y)$	$\rho(Y_1, Y_2)$
0.20	0.12	0.08	**0.70**	**0.918**	2.21	1.51	**0.73**	**0.924**
6 $(= \eta)$	0.09	0.07	**1.02**	**-0.101**	2.15	1.47	**1.05**	**-0.061**

D.3 Liquidity effect and exchange rates dynamics in the model with Taylor rule

Figure 4 displays the IRF of the exchange rates and interest rates when there is a 1% decrease in the domestic interest rate rule in period 1. Consistent with our previous analysis, we consider here an expansionary monetary policy.

Figure 4: In the model with an interest rate rule



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